Answer on Question #48165 – Math – Integral Calculus

Question: evaluate the definite integrals listed below

$$\int_{e}^{6} \frac{\ln(x)}{x}\, dx$$

$$\int_{-1}^{0} \frac{x}{x+2}\, dx$$

$$\int_{0}^{3} x \cdot e^{2x^2}\, dx$$

Solution:

1) Let us change the variable of integration:

$$\int_{e}^{6} \frac{\ln(x)}{x}\, dx = \int_{e}^{6} \ln(x)\, d(\ln(x)) = \frac{\ln^2(x)}{2}\bigg|_{e}^{6} = \frac{\ln^2(6)}{2} - \frac{\ln^2(e)}{2} = \frac{\ln^2(6)}{2} - \frac{1}{2}$$

2) In this case let us use the following trick:

$$\int_{-1}^{0} \frac{x}{x+2}\, dx = \int_{-1}^{0} \frac{x+2-2}{x+2} dx = \int_{-1}^{0} \left(1 - \frac{2}{x+2}\right) dx = (x - 2\ln(x+2))|^{0}{}_{-1}$$
$$= -2\ln(2) - (-1 - 2\ln(1)) = 1 - 2\ln(2)$$

3) Here we are also going to change the variable of integration:

$$\int_{0}^{3} x \cdot e^{2x^2}\, dx = \frac{1}{2}\int_{0}^{3} e^{2x^2} dx^2 = \frac{1}{4}\int_{0}^{3} e^{2x^2} d(2x^2) = \frac{1}{4} e^{2x^2}\bigg|_{0}^{3} = \frac{1}{4}(e^{18} - 1)$$

Answer:

$$\int_{e}^{6} \frac{\ln(x)}{x}\, dx = \frac{\ln^2(6)}{2} - \frac{1}{2}$$

$$\int_{-1}^{0} \frac{x}{x+2}\, dx = 1 - 2\ln(2)$$

$$\int_{0}^{3} x \cdot e^{2x^2}\, dx = \frac{1}{4}(e^{18} - 1)$$