Answer on Question #48163 – Math – Integral Calculus

a) $\int e^x\left(e^{3x}-5\right)dx.$

Solution.

$$\int e^x\left(e^{3x}-5\right)dx=\int\left(e^x\cdot e^{3x}-e^x\cdot 5\right)dx=\int\left(e^{4x}-5e^x\right)dx=\int e^{4x}dx-5\int e^x dx.$$

$$\int e^{4x}dx=\begin{bmatrix} t=4x \\ dt=4dx\Rightarrow dx=\dfrac{dt}{4}\end{bmatrix}=\int\frac{e^t}{4}dt=\frac{e^t}{4}+C=\frac{1}{4}e^{4x}+C;$$

$$\int e^x dx=e^x+C$$

Hence,

$$\int e^x\left(e^{3x}-5\right)dx=\frac{1}{4}e^{4x}-5e^x+C.$$

Answer: $\frac{1}{4}e^{4x}-5e^x+C.$

b) $\int x\left(\sqrt{2x^2+3}\right)dx.$

Solution.

$$\int x\left(\sqrt{2x^2+3}\right)dx=\begin{bmatrix} t=2x^2+3 \\ dt=\left(2x^2+3\right)'dx=4xdx \\ xdx=\dfrac{1}{4}dt\end{bmatrix}=\int\frac{1}{4}\sqrt{t}dt=\frac{1}{4}\int t^{\frac{1}{2}}dt=$$

$$=\frac{1}{4}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{4}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{2}{4}\cdot\frac{\left(2x^2+3\right)^{\frac{3}{2}}}{3}+C=\frac{\left(2x^2+3\right)^{\frac{3}{2}}}{6}+C.$$

Answer: $\frac{\left(2x^2+3\right)^{\frac{3}{2}}}{6}+C.$

c) $\int\frac{x}{\sqrt{3-x}}dx.$

Solution.

$$\int \frac{x}{\sqrt{3-x}}dx = \int \frac{x-3+3}{\sqrt{3-x}}dx = \int \frac{-(3-x)}{\sqrt{3-x}}dx + \int \frac{3}{\sqrt{3-x}}dx = -\int \sqrt{3-x}dx + 3\int \frac{dx}{\sqrt{3-x}}.$$

$$\int \sqrt{3-x}dx = \begin{bmatrix} t = 3-x \\ dt = -dx \end{bmatrix} = -\int \sqrt{t}dt = -\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = -\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{2(3-x)^{\frac{3}{2}}}{3} + C;$$

$$\int \frac{dx}{\sqrt{3-x}} = \begin{bmatrix} t = 3-x \\ dt = -dx \end{bmatrix} = -\int \frac{dt}{\sqrt{t}} = -\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = -\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = -2\sqrt{3-x} + C$$

Hence,

$$\int \frac{x}{\sqrt{3-x}}dx = \frac{2(3-x)^{\frac{3}{2}}}{3} + 3\left(-2\sqrt{3-x} + C\right) = \frac{2}{3}\sqrt{(3-x)^3} - 6\sqrt{3-x} + C$$

Answer: $\frac{2}{3}\sqrt{(3-x)^3} - 6\sqrt{3-x} + C$.