

Answer on Question #48126 – Math – Trigonometry

$$\frac{\cos(2a)\cos(3a) - \cos(2a)\cos(7a) + \cos(a)\cos(10a)}{\sin(4a)\sin(3a) - \sin(2a)\sin(5a) + \sin(4a)\sin(7a)} = \cot(6a)\cot(5a)$$

Solution:

We start with the original expression.

$$\frac{\cos(2a)\cos(3a) - \cos(2a)\cos(7a) + \cos(a)\cos(10a)}{\sin(4a)\sin(3a) - \sin(2a)\sin(5a) + \sin(4a)\sin(7a)}$$

We can simplify the denominator and numerator.

$$\begin{aligned} & \frac{\cos(2a)\cos(3a) - \cos(2a)\cos(7a) + \cos(a)\cos(10a)}{\sin(4a)\sin(3a) - \sin(2a)\sin(5a) + \sin(4a)\sin(7a)} = \\ & = \frac{\cos(2a)(\cos(3a) - \cos(7a)) + \cos(a)\cos(10a)}{\sin(4a)(\sin(3a) + \sin(7a)) - \sin(2a)\sin(5a)} = \\ & = \frac{\cos(2a)(-2\sin(5a)\sin(2a)) + \cos(a)\cos(10a)}{\sin(4a)(-2\sin(5a)\sin(2a)) - \sin(2a)\sin(5a)} = \\ & = \frac{\cos(2a)(-2\sin(5a)\sin(2a)) + \cos(a)\cos(10a)}{\sin(5a)\sin(2a)(-2\sin(4a) - 1)} = \frac{\cos(a) + \cos(11a)}{\cos(a) - \cos(11a)} = \\ & = \frac{-2\cos(6a)\cos(5a)}{-2\sin(6a)\sin(5a)} = \frac{\cos(6a)\cos(5a)}{\sin(6a)\sin(5a)} = \cot(6a)\cot(5a) \end{aligned}$$

Thus we can conclude that

$$\frac{\cos(2a)\cos(3a) - \cos(2a)\cos(7a) + \cos(a)\cos(10a)}{\sin(4a)\sin(3a) - \sin(2a)\sin(5a) + \sin(4a)\sin(7a)} = \cot(6a)\cot(5a)$$