Answer on Question #48111 – Math - Differential Calculus | Equations

Find one set of values of the numbers a and b, such that the function $y = e^{ax} + b\ln(x)$ is a solution of the differential equations given below.

a. $y'' - ay' = 0$

b. $y' + y - \left(\frac{b}{x}\right) = 0$

Solution

$$y = e^{ax} + b\ln(x)\,;\; y' = ae^{ax} + \left(\frac{b}{x}\right);\; y'' = (a^2)e^{ax} - \left(\frac{b}{x^2}\right).$$

a. $y'' - ay' = 0$

$$(a^2)e^{ax} - \left(\frac{b}{x^2}\right) - (a)\left(ae^{ax} + \left(\frac{b}{x}\right)\right) = 0$$

$$(a^2)e^{ax} - \left(\frac{b}{x^2}\right) - (a^2)e^{ax} - \left(\frac{ab}{x}\right) = 0$$

$$\left(\frac{b}{x}\right)\left(-\frac{1}{x} - a\right) = 0$$

Note that when $b = 0$, the above equation obviously holds true regardless of value of x (unless x is also zero). a can be any value, so we can also set it to be 0.

Thus, we have: $a = 0$ and $b = 0$.

b. $y' + y - \left(\frac{b}{x}\right) = 0$

$$ae^{ax} + \left(\frac{b}{x}\right) + e^{ax} + b\ln(x) - \left(\frac{b}{x}\right) = 0$$

$$ae^{ax} + e^{ax} + b\ln(x) = 0$$

Once again, we can let $b = 0$.

However, if we let $a = 0$, e^{ax} will be equal to 1, which means that the equation will not be balanced. Thus, we should let $a = -1$ instead, so that we will have $-e^{-x} + e^{-x}$, which will result in 0.

Hence, $a = -1, b = 0$.