Answer on Question #48110 – Math – Calculus

Question:

Find the absolute maximum and minimum of the function

$$f(x) = \frac{x}{x^2 + 4}$$

on the interval $[0, 4]$.

Solution:

To find the absolute maximum and minimum of some function, we have to find extreme points of this functions that lie inside the defined interval and then calculate values of the function at these points. Besides, we have to check the values of the function at the ends of the interval. Extreme points x_0 can be found using the following condition

$$f'(x_0) = 0$$

The derivative of $f(x)$ is

$$f'(x) = \left(\frac{x}{x^2 + 4}\right)' = \frac{1}{x^2 + 4} - \frac{2x^2}{(x^2 + 4)^2} = \frac{4 - x^2}{(x^2 + 4)^2}$$

Condition $f'(x_0) = 0$ implies that

$$4 - x_0^2 = 0 \Rightarrow x_0 = \pm 2$$

Only point $x_0 = 2$ lies inside the interval $[0, 4]$. Next, we have to check if this point is maximum or minimum point. If $f''(x_0) > 0$ then there is minimum of the function at x_0 and if $f''(x_0) < 0$ then there is maximum of the function at x_0.

$$f''(x) = -\frac{2x}{(x^2 + 4)^2} - \frac{4x(4 - x^2)}{(x^2 + 4)^3} = -2x \cdot \frac{12 - x^2}{(x^2 + 4)^3}$$

$$f''(2) = -\frac{1}{16} < 0$$

Therefore, we have maximum point at point $x_0 = 2$ and $f(2) = \frac{1}{4}$.

Let us calculate values of the function at the ends of the given interval now:

$$f(0) = 0$$

$$f(4) = \frac{1}{5} < f(2)$$

Thus we have maximum of the function at the point $x = 2$

$$f(2) = \frac{1}{4}$$

and minimum of the function at the point $x = 0$

$$f(0) = 0$$

Answer:

$$f(2) = \frac{1}{4} - \text{maximum}$$

$$f(0) = 0 - \text{minimum}$$