Answer on Question #48103 – Math – Calculus

Sketch one graph of a single function f that satisfies all of the conditions below:

a. domain is $(0,\infty)$;

b. $\lim\limits_{x\to 0_+} f(x) = \infty$;

c. $\lim\limits_{x\to\infty} f(x) = 2$;

d. $f'(x) < 0$ on the interval $(0,3)$;

e. $f'(x) > 0$ on the interval $(3,\infty)$;

f. $f'(3) = 0$;

g. $f''(x) > 0$ on the interval $(0,6)$

h. $f''(x) < 0$ on the interval $(6,\infty)$.

Solution.

By a, domain of function $f(x)$ is $(0,\infty)$, therefore $x > 0$.

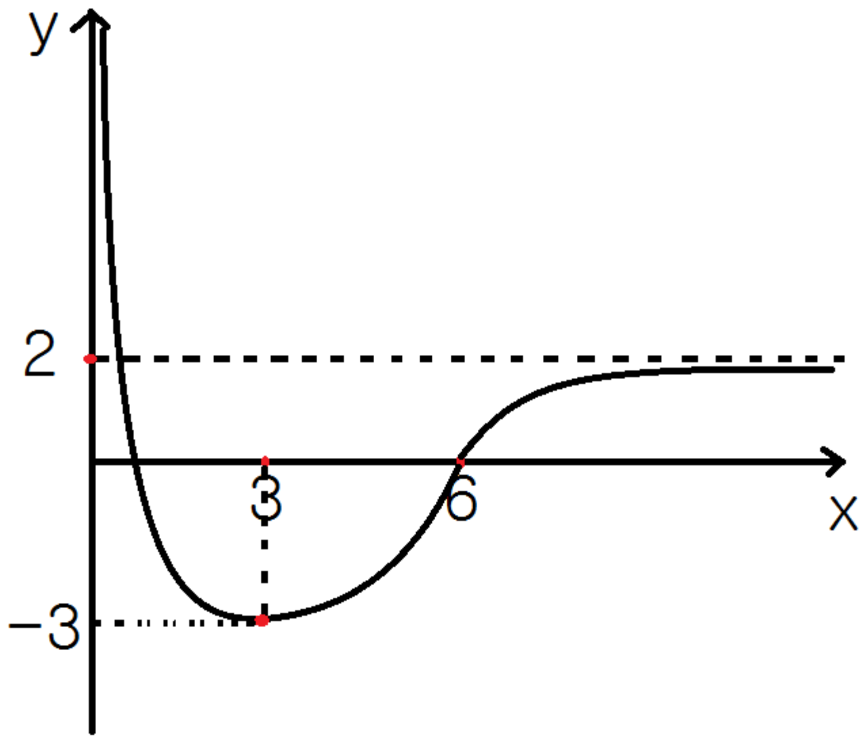
By d, $\forall x \in (0,3)$: $f'(x) < 0 \Longrightarrow f$ decreases on $(0,3)$. By b, $\lim\limits_{x\to 0_+} f(x) = +\infty$.

By c and e,

$$\begin{cases} \lim\limits_{x\to\infty} f(x) = 2 \\ \forall x \in (3,+\infty): f'(x) > 0 \end{cases} \Longrightarrow \forall\, x \in (3,\infty): f(x) < 2;\ f \text{ increases on } (3,\infty).$$

Take into account d, e, f, so $f'(x) < 0$ on $(0,3)$; $f'(x) > 0$ on $(3,\infty)$; $f'(3) = 0 \Longrightarrow x = 3$ is a point of local minimum (in this problem it will be global minimum). Note that by g and h, $f''(x) > 0$ on $(0,6)$, $f''(x) < 0$ on $(6,\infty) \Longrightarrow$ the graph of function is convex downward on $(0,6)$, the graph of function is convex upward on $(6,\infty)$.

So, the graph of $f(x)$ can be like this:


y
2
3
6
x
−3