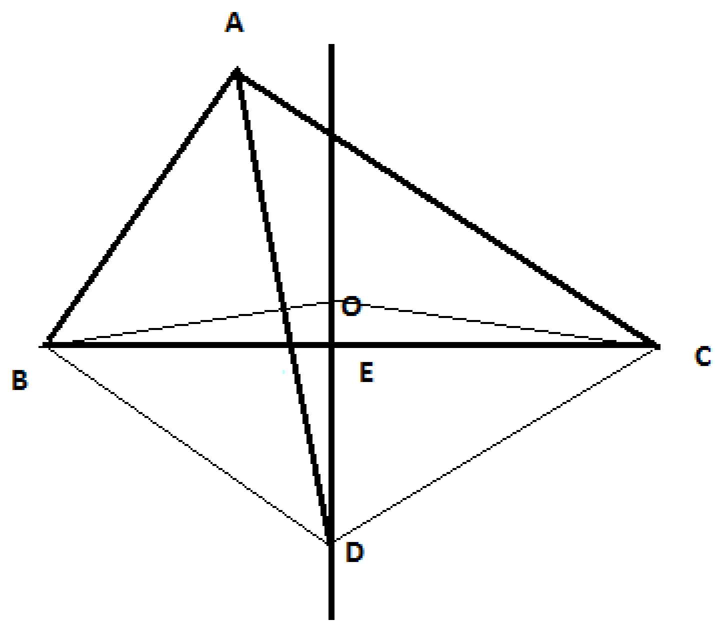
Answer on Question #47912 – Math – Geometry

In any triangle ABC,the angle bisector of angle A and perpendicular bisector of BC intersect. Prove that they intersect on the circumcircle of the triangle ABC

Solution.



Let perpendicular bisector of side BC and angle bisector of ∠A meet at point D. Let the perpendicular bisector of side BC intersect it at E.

Perpendicular bisector of side BC will pass through circumcenter O of the circle. ∠BOC and ∠BAC are the angles subtended by arc BC at the center and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

∠BOC = 2 ∠BAC = 2 ∠A (1)

In ΔBOE and ΔCOE,

OE = OE (Common)

OB = OC (Radii of the same circle)

∠OEB = ∠OEC (Each 90° as OD ⊥ BC)

∴ ΔBOE ≅ ΔCOE (RHS congruence rule)

∠BOE = ∠COE (By CPCT) **(2)**

However, ∠BOE + ∠COE = ∠BOC

⇒ ∠BOE +∠BOE = 2 ∠A [Using equations (1) and (2)]

⇒ 2 ∠BOE = 2 ∠A

⇒ ∠BOE = ∠A

∴ ∠BOE = ∠COE = ∠A

The perpendicular bisector of side BC and angle bisector of ∠A meet at point D.

∴ ∠BOD = ∠BOE = ∠A **(3)**

Since AD is the bisector of angle ∠A,

⇒ 2 ∠BAD = ∠A **(4)**

From equations (3) and (4), we obtain

∠BOD = 2 ∠BAD

This can be possible only when point BD will be a chord of the circle. For this,

the point D lies on the circumcircle.

Therefore, the perpendicular bisector of side BC and the angle bisector

of ∠A meet on the circumcircle of triangle ABC.