Answer on Question #47781 – Math – Geometry

In triangle ABC m(<A)=30. M(<C)=80, a+2b=15 cm. Find the value of c and the length of the radius of the circle

Solution:

Given:

$\angle A = 30°$

$\angle C = 80°$

$a + 2b = 15\ ; \quad a = 15 - 2b \qquad (1)$

$c, R - ?$

We can find the angle $\angle B$ of the triangle:

$$\angle A + \angle B + \angle C = 180°$$

$$\angle B = 180° - \angle A - \angle C = 180° - 80° - 30° = 70°$$

Law of sines for the triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad (2)$$

Plug(1) into (2):

$$\frac{15 - 2b}{\sin A} = \frac{b}{\sin B}$$

$$(15 - 2b)\sin B = b \sin A$$

$$15 \cdot \sin B = b(\sin A + 2\sin B)$$

$$b = \frac{15 \cdot \sin B}{\sin A + 2\sin B} = \frac{15\ \text{cm} \cdot \sin 70°}{\sin 30° + 2\sin 70°} = 5.9\ \text{cm}$$

$$a = 15\text{cm} - 2 \cdot 5.9\ \text{cm} = 3.2\ \text{cm}$$

Law of sines for the triangle ABC:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c = \frac{b \sin C}{\sin B} = \frac{5.9\ \text{cm} \cdot \sin 80°}{\sin 70°} = 6.2\ \text{cm}$$

Length of the radius of the circle Inscribed within a triangle ABC.

$$R = \frac{\text{Triangle area}}{k} = \frac{\sqrt{k(k-a)(k-b)(k-c)}}{k},$$

$$\text{where } k = \frac{1}{2}(a + b + c) = \frac{1}{2}(3.2\ \text{cm} + 5.9\ \text{cm} + 6.2\ \text{cm}) = 7.65\ \text{cm}$$

$$R = \frac{\sqrt{7.65\ \text{cm}(7.65\ \text{cm} - 3.2\ \text{cm})(7.65\ \text{cm} - 5.9\ \text{cm})(7.65\ \text{cm} - 6.2\ \text{cm})}}{7.65\ \text{cm}} = 1.2\ \text{cm}$$

Answer: $c = 6.2\ \text{cm}; R = 1.2\ \text{cm}$