Answer on Question #47686 – Math – Differential Calculus | Equations

Please solve the following:

1. y''=1+(y')^2

2. xdy-(3y+x^5*y^1/3)dx=0

Solution.

1. $y'' = 1 + (y')^2$

$$v(x) = y' \to \ v' = 1 + v^2 \to \frac{dv}{1+v^2} = dx \to \ arctan(v) = x + c_1$$

$$\to arctan(y') = x + c_1 \to$$

$$y' = tan(x + c_1) \to \ y = \int tan(x + c_1)dx = -ln[cos(x + c_1)] + c_2.$$

$$y = -ln[cos(x + c_1)] + c_2 \ .$$

2. $xdy - \left(3y + x^5 y^{\frac{1}{3}}\right) dx = 0$

Verify that $y = 0$ is the solution to equation. Let $y \neq 0$ and

$$xy' - 3y = x^5 y^{\frac{1}{3}} \to \frac{y'}{y^{\frac{1}{3}}} - \frac{3}{x} y^{\frac{2}{3}} = x^4 \to \left(y^{\frac{2}{3}}\right)' = \frac{2}{3y^{\frac{1}{3}}} y' \to$$

$$\to \frac{2y'}{3y^{\frac{1}{3}}} - \frac{2}{x} y^{\frac{2}{3}} = \frac{2}{3} x^4 \to$$

$$v(x) = y^{\frac{2}{3}} \to \ v' - \frac{2v}{x} = \frac{2x^4}{3} \to \ \frac{v'}{x^2} - \frac{2v}{x^3} = \frac{2x^2}{3} \to \ \frac{d}{dx}\left(\frac{v}{x^2}\right) = \frac{2x^2}{3} \to$$

$$\to \ \frac{v}{x^2} = \frac{2x^3}{9} + c \ \to \ y^{\frac{2}{3}} = \frac{2x^5}{9} + cx^2 \to \ y = \frac{x^3}{27} (2x^3 + 9c)^{\frac{3}{2}}$$

$y = \frac{x^3}{27} (2x^3 + 9c)^{\frac{3}{2}}$ **and** $y = 0$ **.**