Answer on Question #47588 – Math – Calculus

The diagram shows part of the curve y=2-18/(2x+3), which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is 9x+4y=27.

(ii) Find the length of BC.

Solution:

(i) The diagram shows part of the curve y=2-18/(2x+3), which crosses the x-axis at A and the y-axis at B. Find the points A and B. For point A we have, y=0. Hence

$$2-\frac{18}{2x+3}=0 \Longrightarrow 2x+3=9 \Longrightarrow 2x=6 \Longrightarrow \mathbf{x=3}.$$

For point B we have, x=0 and

$$y=2-\frac{18}{2*0+3} \Longrightarrow \mathbf{y=-4}.$$

We found coordinates A(3; 0) and B (0; -4).

Next, the equation of the normal line to y = f(x) at $(\boldsymbol{x_0}, \boldsymbol{y_0})$ is:

$$y=\boldsymbol{y_0} - \frac{\mathbf{1}}{\boldsymbol{f'(x_0)}}(\boldsymbol{x}-\boldsymbol{x_0}).$$

For

$$y=2-\frac{18}{2x+3}$$

we have

$$y'=f'(x)=\frac{36}{(2x+3)^2}.$$

Therefore the equation of the normal line to $y=2-\frac{18}{2x+3}$ at point A(3; 0) is

$$y=\mathbf{0}-\frac{(2*3+3)^2}{\mathbf{36}}(\boldsymbol{x}-\mathbf{3})=-\frac{\mathbf{9}}{\mathbf{4}}(\boldsymbol{x}-\mathbf{3}).$$

or the same $\mathbf{4y+9x=27}$.

We have shown that the equation of the line AC is $\mathbf{4y+9x=27}$.

(ii) Next, we find coordinates of point C. The line AC crosses the y-axis at C, hence x=0 and

$$4y+9*0=27 \Longrightarrow y=\frac{27}{4}$$

Point C has coordinates (0, $\frac{\mathbf{27}}{\mathbf{4}}$). The length of BC is equal distance between points B and C:

$$|BC|=\sqrt{(0-0)^2+(-4-\frac{27}{4})^2}=\left|-4-\frac{27}{4}\right|=\frac{43}{4}=10{,}75.$$

The length of BC is equal $\mathbf{10.75}$.