

Answer on Question #47534 – Math – Geometry

How to prove the volume formula of cone is $\frac{1}{3}\pi r^2 h$; without using integration?

Solution.

Suppose we take a slice of the pyramid with the cone inside, from some way up the pyramid. This will look like a square with a circle fitting inside. Radius of the cone at this point, will be x .

The area of the circle is πx^2

The area of the square is $2x \times 2x = 4x^2$

The ratio of the circle to the square is $\frac{\pi}{4}$.

The same is true for every slice we take: the area of the circle is $\frac{\pi}{4}$ of the area of the square.

So, the volume of the cone will be $\frac{\pi}{4}$ the volume of the pyramid.

The pyramid's volume is $\frac{4r^2 h}{3}$.

So the cone's volume is $\frac{4r^2 h}{3} * \frac{\pi}{4} = \frac{\pi r^2 h}{3}$.