Answer on Question #47438 – Math – Calculus

1. Let A={1,2,3,4} and B={a,b,c}.

(i) Write down a function from A to B

(ii) Write down a different function from A to B

(iii) Write down a relation from A to B which is not a function

(iv) How many functions are there from A to B?

2. Determine whether each of the statements that follow are true or false. If the statement is true, give a reason. If it is false, give a counterexample (i.e. a function for which the statement is false)

(i) The domain of every function is a subset of R.

(ii) If f:R=>R is a function and xER then f(2x) = 2f(x)

(iii) If h:A=>B is a function and a, bEA then h(a) = h(b) implies that a=b

Solution.

1. **A relation is a set of ordered pairs (x, y).**

A function is a relation that does not contain two pairs with the same first component.

(i) $\{(1,a),(2,b),(3,c),(4,a)\}$
(ii) $\{(1,a),(2,a),(3,a),(4,a)\}$
(iii) $\{(1,a),(1,b),(2,b),(3,c),(4,a)\}$
(iv) $For\ each\ element\ from\ A\ we\ can\ choose\ \ each\ element\ from\ B.$ **If function $f: A \to B$, $A = \{a_1, a_2, a_3, a_4\}$, then we count the number of possible functions from A to B, $a_1 \to |B|, a_2 \to |B|, a_3 \to |B|$, $a_4 \to |B|$, i.e., this number is $|B| \cdot |B| \cdot |B| \cdot |B| = |B|^{|A|} = 3^4 = 81.$**

$$Thus, there\ are\ 81\ different\ functions\ from\ A\ to\ B.$$

2.

(i) False. Set A can be a set of arbitrary elements, for ex. “red”, “blue”, “black”.

(ii) False. For ex. $f(x) = x^2 \rightarrow \ f(2x) = 4f(x) \neq 2f(x)$. .

(iii) False. For ex. $f(x) = sin(x)$. $f(a + 2\pi) = f(a)$ $but\ a \neq a + 2\pi$.