Question #4736A random sample of 25 was drawn from a normal distribution with a standard deviation of 5. The sample mean is 80. Determine the 95% confidence interval estimate of the population mean.

Solution. Let $\{\xi_n\}_{n=1,25}$ be our sample drawn from normal distribution.Condition implies $\xi_1 \simeq \mathcal{N}(m, 25)$. We are to determine the 95% confidence interval estimate of m. Let find $\mathsf{P}\left(\left|\frac{\frac{1}{25}\sum_{k=1}^{25}\xi_k - m}{5\cdot 1/5}\right| \le x_{0.95}\right) = 0.95$. It is evident that $\zeta = \frac{\frac{1}{25}\sum_{k=1}^{25}\xi_k - m}{5\cdot 1/5} \simeq \mathcal{N}(0,1)$. Now determine $x_{0.95}$ from relation $\mathsf{P}(|\zeta| < x_{0.95}) = 0.95$, hence $\Phi(x_{0.95}) = \dfrac{1+0.95}{2} = 0.975$, thus $x_{0.95} \approxeq 1.96$. And the confidence interval is $(80 - 1.96, 80 + 1.96) = (78.4, 81.96)$.

Answer. $(78.4, 81.96)$.