Answer on Question #47192 – Math – Integral Calculus

Find the surface area of the band of the sphere generated by revolving the arc of the circle $x^2 + y^2 = r^2$ lying above the interval $[-a, a], a < r$.

Solution

In the case when $f(x)$ is positive and has a continuous derivative, the surface area of the surface generated by revolving the curve $y = f(x), x_1 \le x \le x_2$ about the x – axis is

$$S = 2\pi \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx .$$

In our case: $y = \sqrt{r^2 - x^2}, x_1 = -a, x_2 = a,$ $\frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{r^2 - x^2}}$.

Thus,

$$S = 2\pi \int_{-a}^{a} \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}}\, dx = 2\pi \int_{-a}^{a} \sqrt{r^2 - x^2} \frac{r}{\sqrt{r^2 - x^2}}\, dx = 2\pi \int_{-a}^{a} dx = 2\pi r x|_{-a}^{a}.$$

$$S = 2\pi a r - (-2\pi a r) = 4\pi a r.$$

Answer: $\boldsymbol{4\pi a r}$.