Answer on Question #47140 – Mathematics – Calculus

Question:

Evaluate

$$\lim_{x\to 7}\frac{\sqrt{7x}-7}{\sqrt{3x-8}-\sqrt{13}}, \qquad (1)$$

$$\lim_{x\to a}\left(\frac{1}{a-x}-\left(\frac{a}{a^2-x^2}\right)\right), \qquad (2)$$

Solution:

1) In this case the direct substitution “7” for “x” yields

$$\lim_{x\to 7}\frac{\sqrt{7\cdot 7}-7}{\sqrt{3\cdot 7-8}-\sqrt{13}}=\left(\frac{0}{0}\right).$$

We have received the "indeterminate" value. To find the limit we use the L`Hospital`s rule (we apply the first derivative both to the numerator anddenominator):

$$\lim_{x\to 7}\frac{\left(\sqrt{7x}-7\right)}{\left(\sqrt{3x-8}-\sqrt{13}\right)}=\lim_{x\to 7}\frac{\frac{7}{2\sqrt{7x}}}{\frac{3}{2\sqrt{3x-8}}}=\lim_{x\to 7}\frac{7\sqrt{3x-8}}{3\sqrt{7x}}=\frac{7\sqrt{3\cdot 7-8}}{3\sqrt{7\cdot 7}}=\frac{\sqrt{13}}{3}$$

2) Substituting “a” for “x” we obtain

$$\lim_{x\to a}\left(\frac{1}{a-a}-\left(\frac{a}{a^2-a^2}\right)\right)=(\infty-\infty)$$

As we see, it is the "indeterminate" value, but in this case we cannot use directly the L`Hospital`s rule. Therefore first we rewrite the fraction in the following form:

$$\lim_{x\to a}\frac{\left(\frac{1}{\frac{a}{a^2-x^2}}-\frac{1}{\frac{1}{a-x}}\right)}{\left(\frac{1}{\frac{1}{a-x}\cdot\frac{a}{a^2-x^2}}\right)}.$$

The direct substitution “a” for “x” yields

$$\lim_{x\to a}\frac{\left(\frac{1}{\frac{a}{a^2-x^2}}-\frac{1}{\frac{1}{a-x}}\right)}{\left(\frac{1}{\frac{1}{a-x}\cdot\frac{a}{a^2-x^2}}\right)}=\left(\frac{0}{0}\right).$$

Now we can apply the L`Hospital`s rule:

$$\lim_{x\to a}\frac{\left(\frac{a^2-x^2}{a}-(a-x)\right)}{\left(\frac{(a^2-x^2)\cdot(a-x)}{a}\right)}=\lim_{x\to a}\frac{\left(-\frac{2x}{a}+1\right)}{\left(\frac{(-2x)\cdot(a-x)+(a^2-x^2)\cdot(-1)}{a}\right)}$$

$$=\frac{\left(-\frac{2a}{a}+1\right)}{\left(\frac{(-2a)\cdot(a-a)-(a^2-a^2)}{a}\right)}=\frac{-1}{0}=\infty.$$

Answer. 1) $\lim_{x\to 7}\frac{\sqrt{7x}-7}{\sqrt{3x-8}-\sqrt{13}}=\frac{\sqrt{13}}{3}$, 2) $\lim_{x\to a}\left(\frac{1}{a-x}-\left(\frac{a}{a^2-x^2}\right)\right)=\infty$.