Answer on Question #47046 – Math - Set Theory

Problem.

Prove that if A and B are any two sets such that A is proper subset of B, then A union B=B-

1) by direct method

2) by proving its contrapositive.

3) by contradiction.

Solution.

Consider 2 sets A, B .

As $A \subset B$ then $\forall x[x \in A \Rightarrow x \in B]$.

As A is a proper subset of B then $\exists x[x \in B \wedge x \notin A]$.

1) by direct method

$P \Rightarrow Q$.

$\forall x[x \in A \cup B \Rightarrow x \in A \vee x \in B \Rightarrow x \in B \vee x \in B \Rightarrow x \in B]$.

Vice versa, $\forall x[x \in B \Rightarrow x \in A \vee x \in B \Rightarrow x \in A \cup B]$. The proof is complete.

2) by proving its contrapositive$\neg Q \Rightarrow \neg P$.

Obviously, $A \cup B \supseteq B$. Therefore, $A \cup B \neq B \Rightarrow \exists x'[x' \in A \cup B \wedge x' \notin B]$.

So, $\exists x'[x' \in A \cup B \wedge x' \notin B \Rightarrow (x' \in A \vee x' \in B) \wedge x' \notin B \Rightarrow (x' \in A \wedge x' \notin B) \vee (x' \in B \wedge x' \notin B) \Rightarrow$

$(x' \in A \wedge x' \notin B)]$, which is the negation to $\forall x[x \in A \Rightarrow x \in B]$ - in other words, the negation to $A \subset B$ - and the proof is complete.

3) by contradiction $P \wedge \neg Q$.

To prove the proposition given, we consider the opposite - $P \wedge \neg Q$. Remember that in the second part we proved that $\neg Q \Rightarrow \neg P$. That's why $P \wedge \neg Q \Rightarrow P \wedge \neg P \equiv \bot$ which arrives at the contradiction we want. The proof is complete.