Question. Discuss the continuity of the function f, defined by

$$f(x) = \begin{cases} x^2 - 1, & x \le 1, \\ 1 - 1/x, & x \ge 1. \end{cases}$$

Solution. 1) Suppose $x < 1$. Then $f(x) = x^2 - 1$ is a polynomial, and therefore it is continuous at each such x.

2) Suppose $x = 1$. Then left limit of f at $x = 1$ is equal to

$$\lim_{x \to 1-0} f(x) = \lim_{x \to 1-0} x^2 - 1 = 1^2 - 1 = 1 - 1 = 0,$$

and the right limit is

$$\lim_{x \to 1+0} f(x) = \lim_{x \to 1+0} 1 - 1/x = 1 - 1/1 = 1 - 1 = 0.$$

Thus left and right limits of f at $x = 1$ coincide, and therefore f is continuous at $x = 1$.

3) Finally, let $x > 1$. Then $f(x) = 1 - 1/x$. Since $x \ne 0$, this function is continuous at all such x.

Thus f is continuous at all $x \in \mathbb{R}$.

Answer. f is continuous at all $x \in \mathbb{R}$.