Answer on Question #46919 – Math – Statistics and Probability

If X & Y are symmetric random variables, then show that $E\left(\frac{X}{X+Y}\right)=\frac{1}{2}$.

Solution

Rename X as Y and Y as X. Then, by symmetry,

$$E\left(\frac{X}{X+Y}\right) = E\left(\frac{Y}{Y+X}\right) = E\left(\frac{Y}{X+Y}\right) \qquad (1).$$

Now,

$$E\left(\frac{X+Y}{X+Y}\right) \equiv 1.$$

But the left side is

$E\left(\frac{X}{X+Y}\right)+E\left(\frac{Y}{X+Y}\right)=2E\left(\frac{X}{X+Y}\right)$, by (1) above.

Then $2E\left(\frac{X}{X+Y}\right)=1$. Therefore,

$$E\left(\frac{X}{X+Y}\right)=\frac{1}{2},$$

as was to be shown.