

Answer on Question #46883 – Math – Vector Calculus

1. Given that  $\mathbf{a} = 5\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ . Find  $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})$ .
- a)  $2\mathbf{i} - 12\mathbf{j} - 34\mathbf{k}$
  - b)  $2\mathbf{i} + 12\mathbf{j} + 34\mathbf{k}$
  - c)  $2\mathbf{i} - 3\mathbf{j} + 12\mathbf{k}$
  - d)  $2\mathbf{i} + 2\mathbf{k}$

**Remark.**

We know, that cross product of identical vectors equal zero. For example,  $\mathbf{n} \times \mathbf{n} = 0$ . In our case we have the same situation. But we have no option “zero”. I think the condition of question is a little wrong — it must be: find  $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})$ .

**Solution.**

We know, that cross product  $\mathbf{n} \times \mathbf{m} = (n_y m_z - n_z m_y)\mathbf{i} + (n_z m_x - n_x m_z)\mathbf{j} + (n_x m_y - n_y m_x)\mathbf{k}$ .

Now we must find  $(\mathbf{a} + \mathbf{b})$  and  $(\mathbf{a} - \mathbf{b})$ :

$$\mathbf{a} + \mathbf{b} = (5 + 1)\mathbf{i} + (2 - 3)\mathbf{j} + (-1 + 1)\mathbf{k} = 6\mathbf{i} - \mathbf{j};$$

$$\mathbf{a} - \mathbf{b} = (5 - 1)\mathbf{i} + (2 + 3)\mathbf{j} + (-1 - 1)\mathbf{k} = 4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}.$$

And then we can find  $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})$ :

$$\begin{aligned}(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -1 & 0 \\ 4 & 5 & -2 \end{vmatrix} = \\ &= ((-1)(-2) - 0(5))\mathbf{i} + (0 \cdot 4 - 6(-2))\mathbf{j} + (6 \cdot 5 - (-1)4)\mathbf{k} = \\ &= 2\mathbf{i} + 12\mathbf{j} + 34\mathbf{k}.\end{aligned}$$

**Answer:**

- b)  $2\mathbf{i} + 12\mathbf{j} + 34\mathbf{k}$  is correct.