

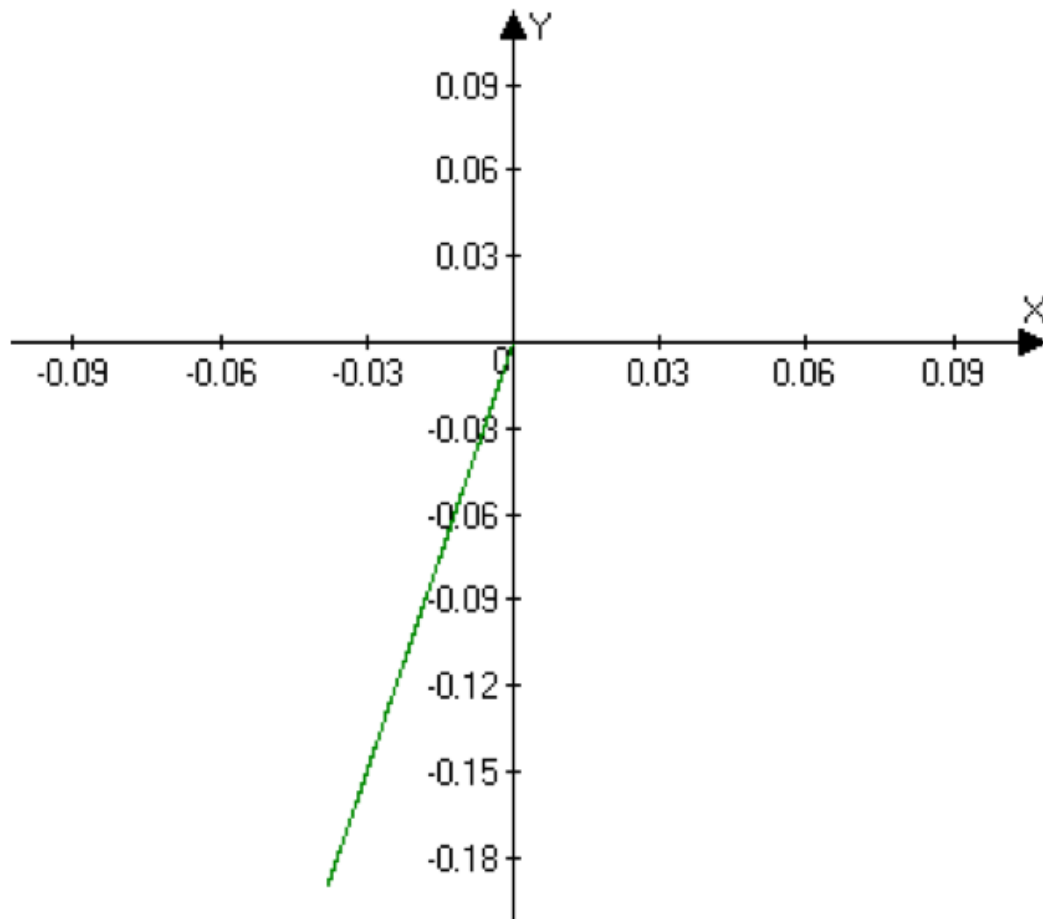
Answer on Question #46860 – Math – Complex Analysis

Obtain the geometric, polar and exponential representations of $(i5 - 1)^{-1}$.

Solution:

$$z = \frac{1}{5i - 1} = \frac{5i + 1}{(5i - 1)(5i + 1)} = \frac{5i + 1}{-25 - 1} = \frac{5i + 1}{-26} = -\frac{1}{26} - \frac{5}{26}i$$

Geometric representation = $x + yi$



Polar representation:

$$z = r(\cos \varphi + i \sin \varphi), r = |z|$$

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(-\frac{1}{26}\right)^2 + \left(-\frac{5}{26}\right)^2} = \sqrt{\frac{1}{676} + \frac{25}{676}} = \sqrt{\frac{26}{676}} = \sqrt{\frac{1}{26}}$$

$$z = \sqrt{\frac{1}{26}} \left(-\sqrt{\frac{1}{26}} + i \left(\sqrt{\frac{25}{26}} \right) \right)$$

$$\varphi = \operatorname{atan}\left(\frac{-\frac{5}{26}}{-\frac{1}{26}}\right) - \pi = \operatorname{atan} 5 - \pi$$

$$z = \sqrt{\frac{1}{26}} (\cos(\tan^{-1} 5 - \pi) + i \sin(\tan^{-1} 5 - \pi))$$

Exponential representation:

$$z = r e^{i\phi}$$

$$z = \sqrt{\frac{1}{26}} e^{i(\tan^{-1} 5 - \pi)}$$