Answer on Question #46855 – Math – Calculus

FIND THE AREA OF THE REGION ENCLOSED BY THE CURVES X^2=Y AND Y=1/2(X^4+X)

Solution:

Find the points of intersection of the curves $\boldsymbol{x^2 = y}$ and $\boldsymbol{y = \frac{1}{2}(x^4 + x)}$. We obtained equation

$$\boldsymbol{x^2 = \frac{1}{2}(x^4 + x).}$$

Solve it

$$x^4 - 2 * x^2 + x = 0$$
$$x * (x^3 - 2 * x + 1) = 0$$
$$x * (x - 1)(x^2 + x - 1) = 0$$

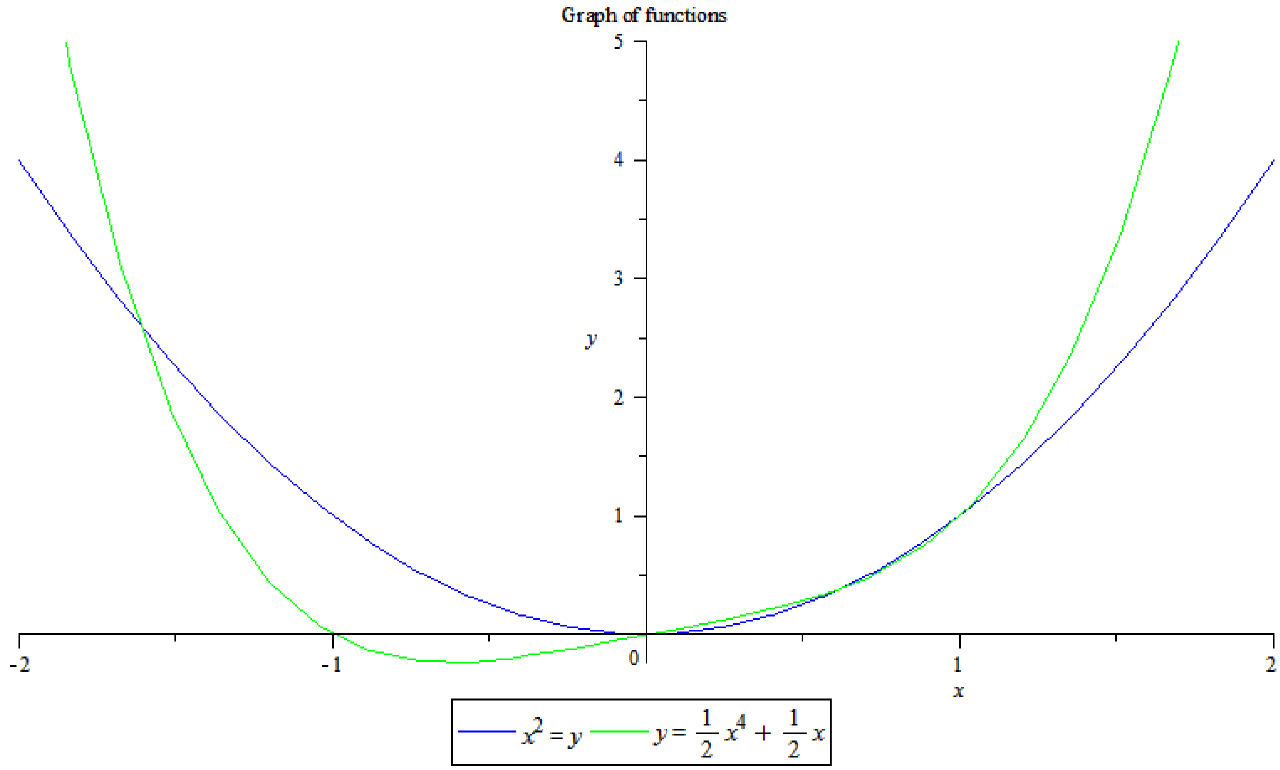
Then $(x^2 + x - 1) = 0$, D=1+4=5 and $x = \frac{-1\pm\sqrt{5}}{2}$.

We have points of intersection of the curves $\boldsymbol{x^2 = y}$ and $\boldsymbol{y = \frac{1}{2}(x^4 + x)}$

$x_1 = \frac{-1-\sqrt{5}}{2}$, x_2 =0, $x_3 = \frac{-1+\sqrt{5}}{2}$, $x_4 = 1$.

We drew the curves $\boldsymbol{x^2 = y}$ and $\boldsymbol{y = \frac{1}{2}(x^4 + x)}$ using Maple15:

```
implicitplot([x^2 = y, y = 1/2*(x^4+x)], x = -2 .. 2, y = -2 .. 5, title = "Graph of functions", color = [blue, green], legend = [x^2 = y, y = 1/2*(x^4+x)])
```



For x in $(\frac{-1-\sqrt{5}}{2}, 0)$ and $(\frac{-1+\sqrt{5}}{2}; 1)$ curve $\boldsymbol{y = x^2}$ is over graph $\boldsymbol{y = \frac{1}{2}(x^4 + x)}$. For x in$(0; \frac{-1+\sqrt{5}}{2})$ curve $\boldsymbol{y = x^2}$ is under graph $\boldsymbol{y = \frac{1}{2}(x^4 + x)}$. Hence, the area of the region enclosed by the curves $\boldsymbol{x^2 = y}$ and $\boldsymbol{y = \frac{1}{2}(x^4 + x)}$ is equal

$$\int_{x_1}^{x_2}\left(x^2 - \frac{1}{2}(x^4 + x)\right)dx + \int_{x_2}^{x_3}\left(-x^2 + \frac{1}{2}(x^4 + x)\right)dx + \int_{x_3}^{x_4}\left(x^2 - \frac{1}{2}(x^4 + x)\right)dx = \left(\frac{x^3}{3} - \frac{1}{2}\left(\frac{x^5}{5} + \frac{x^2}{2}\right)\right)\Big|_{x_0}^{x_1} + \left(-\frac{x^3}{3} + \frac{1}{2}\left(\frac{x^5}{5} + \frac{x^2}{2}\right)\right)\Big|_{x_2}^{x_3} + \left(\frac{x^3}{3} - \frac{1}{2}\left(\frac{x^5}{5} + \frac{x^2}{2}\right)\right)\Big|_{x_3}^{x_4} \approx \mathbf{1}.$$

We use Maple15 to evaluated this integral:

$$> \ x1 := -0.5 - \frac{\sqrt{5}}{2};\ x2 := 0;\ x3 := -0.5 + \frac{\sqrt{5}}{2};\ x4 := 1;$$

$$-0.5 - \frac{1}{2}\sqrt{5}$$

$$0$$

$$-0.5 + \frac{1}{2}\sqrt{5}$$

$$1$$

$$evalf\left(\int_{x1}^{x2} \left(x^2 - \frac{1}{2}\left(x^4 + x\right)\right) dx + \int_{x2}^{x3} \left(-x^2 + \frac{1}{2}\left(x^4 + x\right)\right) dx + \int_{x3}^{x4} \left(x^2 - \frac{1}{2}\left(x^4 + x\right)\right) dx\right)$$

$$0.9924858373$$