Answer on Question #46851 – Math – Differential Calculus |Equations

This problem should be solved using a differential equation:

A person is trying to fill a bathtub with water. Water is flowing into the bathtub from the tap at a constant rate of $k\frac{L}{s}$. However, there is a hole in the bottom of the bathtub and water is flowing out of the bathtub at a rate proportional to the square of the volume of water present in the bathtub. If $V(t)$ is the volume of water (in liters) present in the bathtub at time t (in seconds) and the bathtub initially contains $V(0)$ litters of water.

How can a differential equation of this problem be written? (not solve just writing the equation)

Solution

The conservation equations dictate that the rate of change in volume is given by

$$\frac{dV(t)}{dt} = \dot{V_{in}} - \dot{V_{out}},$$

where $\dot{V_{in}}$ and $\dot{V_{out}}$, are the corresponding inflow and outflow. Note that the units of both magnitudes are $\frac{[V]}{[t]}$, which in your case are $\frac{L}{s}$. So we have $\dot{V_{in}} = k$ and $\dot{V_{out}} = \alpha V^2$, where α is a constant of proportionality (measured in $\frac{S}{L}$). Thus, we come up with the following nonlinear first order ODE:

$$\frac{dV}{dt} + \alpha V^2 = k, V(0) = V_0.$$