Answer on Question 46848, Math, Algebra

$a+b+c=0$ then find

$$\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}$$

It's obvious that

$$\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}=\frac{a^3+b^3+c^3}{abc}$$

Let's find

$$(a+b+c)^3$$

$$(a+b+c)^3=((a+b)+c)^3=(a+b)^3+3(a+b)^2c+3(a+b)c^2+c^3=$$

$$=(a^3+b^3+3a^2b+3ab^2)+3(a^2+2ab+b^2)c+3(a+b)c^2+c^3=$$

$$=a^3+b^3+c^3+3(a^2b+ab^2+a^2c+ac^2+b^2c+bc^2)+6abc$$

As $a+b+c=0$, then $a+b=-c, a+c=-b, b+c=-a$, therefore,

$$(a+b+c)^3=a^3+b^3+c^3+3(a+b)c^2+3(a+c)b^2+3(b+c)a^2+6abc=$$

$$=a^3+b^3+c^3-3c^3-3b^3-3a^3+6abc=-2a^3-2b^3-2c^3+6abc$$

As $a+b+c=0$, then $(a+b+c)^3=0$, therefore

$$6abc=2(a^3+b^3+c^3)\Rightarrow a^3+b^3+c^3=3abc$$

which gives us the answer

$$\frac{a^3+b^3+c^3}{abc}=3$$