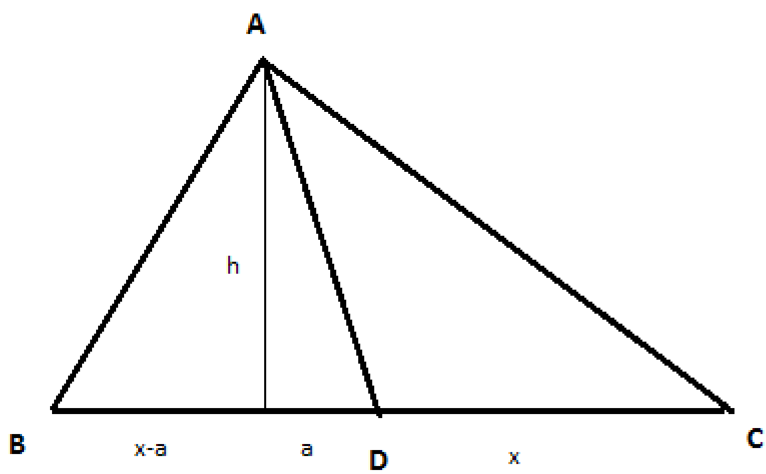
Answer on Question #46827 – Math – Geometry

Q.) AD , BE and CF are medians of the ABC , then prove that

3(AB2 + BC2 + CA2) = 4(AD2 + BE2 + CF2)

Solution.



$$\boldsymbol{AB^2 = h^2 + (x-a)^2,\ \ AC^2 = h^2 + (x+a)^2 \rightarrow}$$

$$\boldsymbol{AB^2 + AC^2 = h^2 + (x-a)^2 + h^2 + (x+a)^2 = 2(h^2 + a^2 + x^2) =}$$

$$\boldsymbol{= 2(h^2 + a^2) + \frac{(2x)^2}{2} = 2AD^2 + \frac{1}{2}BC^2.}$$ **So,**

$$\boldsymbol{AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2} \quad \textbf{(1)}$$

Similarly,

$BC^2 + AC^2 = 2CF^2 + \frac{1}{2}AB^2$ **(2)**

$AB^2 + BC^2 = 2BE^2 + \frac{1}{2}AC^2$ **(3)**

Adding (1), (2), and (3) we have:

$$2(AB^2 + BC^2 + AC^2) = 2(AD^2 + CF^2 + BE^2) + \frac{1}{2}(AB^2 + BC^2 + AC^2)$$

Or,

$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + CF^2 + BE^2)$ **Q.E.D.**