Answer on Question #46816 – Math – Calculus

Test the following series for convergence and divergence :
summation from n=1 to infinity tan^-1(1/n^2+n+1)

Solution:

We have a series

$$\sum_{n=1}^{\infty} \frac{1}{\tan(\frac{1}{n^2+n+1})} \quad (1)$$

Denote $a_n = \frac{1}{\tan(\frac{1}{n^2+n+1})}$

We know $\lim_{x \to 0} \tan x = 0$,

It's mean that $\tan x \sim x$ when $x \to 0$. Hence

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\tan(\frac{1}{n^2+n+1})} = \lim_{n\to\infty}(n^2+n+1) = \infty$$

Because

$$\lim_{n\to\infty} \frac{1}{n^2+n+1} = 0.$$

Therefor series **(1) divergent** as a necessary condition of convergence is not satisfied.

For series $\sum_{n=1}^{\infty} a_n$ we have(necessary condition of convergence):

if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$ and if $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ is divergent.

Actually, maybe it should have been series

$$\sum_{n=1}^{\infty} \tan(\frac{1}{n^2+n+1}) \quad (2)$$

The series **(2) converges.** Because

we have $\tan(\frac{1}{n^2+n+1}) \sim \frac{1}{n^2+n+1}$ when $n \to \infty$. Then $\frac{1}{n^2+n+1} \le \frac{1}{n^2}$ for any natural n. The series

$$\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$$

converges for all $\alpha > 1$ and divergent for all $\alpha \le 1$. Hence series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges and consequently series (2) converge by comparison test:

Comparison test. The terms of the sequence $\{a_n\}$ are compared to those of another sequence $\{b_n\}$. If,

for all n, $0 \le a_n \le b_n$, and $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

However, if for all n, $0 \le b_n \le a_n$, and $\sum_{n=1}^{\infty} b_n$ diverges, then so does $\sum_{n=1}^{\infty} a_n$.