**Answer on Question #46767 – Math – Algorithms | Quantitative Methods**

1.a-Solve $9x^4 - 18x^3 - 31x^2 + 8x + 12 = 0$ by Ferrari's method.

**Solution:**

Solving bi-quadratic polynomials can be done using Ferrari's method, which transforms a bi-quadratic polynomial into a depressed bi-quadratic which has no $x^3$ term.

By substituting $x = y - \frac{b}{4a}$ we arrive at the above equation $y^4 + py^2 + qy + r = 0$

Where $p = \frac{8ac-3b^2}{8a^2}$, $q = \frac{8a^2d+b^3-4abc}{8a^3}$, $r = \frac{16ab^2c-64a^2bd-3b^4+256a^3e}{256a^4}$.

If $q \neq 0$ then we have to solve the auxiliary cubic equation.

$$z^3 + pz^2 + \frac{p^2 - 4r}{4}z - \frac{q^2}{8} = 0$$

Start to find the value of q. In our case we have a=9, b=-18, c=-31, d=8, e=12.

Substitute the given values into the formula for q noted above.

$$q = \frac{8a^2d + b^3 - 4abc}{8a^3} = \frac{8 \cdot (9)^2 \cdot 8 + (-18)^3 - 4(9)(-18)(-31)}{8(9)^3}$$
$$= \frac{5184 - (-5832) - 20088}{5832} = -1.55556$$

If $q \neq 0$ then this equation is always a positive root, which we denote $z_0$. Then the roots of the original equation can be obtained from the formulas.

$$z^3 + pz^2 + \frac{p^2 - 4r}{4}z - \frac{q^2}{8}$$

Find the value of p.

$$p = \frac{8ac - 3b^2}{8a^2} = \frac{8(9)(-31) - 3(-18)^2}{8(9)^2} = \frac{-2232 - 972}{648} = -4.9444$$

Calculate the value of r.

$$r = \frac{16ab^2c - 64a^2bd - 3b^4 + 256a^3e}{256a^4}$$
$$= \frac{16(9)(-18)^2(-31) - 64(9)^2(-18)(8) - 3(-18)^4 + 256(9)^3(12)}{256(9)^4}$$
$$= 0.729167$$

$$r = 0.729167$$

Then we solve the following equation.

$$z^3 + (-4.944)z^2 + \frac{(-4.944)^2 - 4(0.729)}{4}z - \frac{(-1.556)^2}{8} = 0$$

Simplify to find the value of z.

$$z^3 - 4.944z^2 + 5.382z - 0.303 = 0$$

As a result of solution we obtained the following roots.

$$z_1 \approx 0.0595$$

$$z_2 \approx 1.5077$$

$$z_3 \approx 3.37672$$

Now we can find the root of the original equation.

$$y_1 = \frac{\sqrt{2z_0} - \sqrt{2z_0 - 4(\frac{p}{2} + z_0 + \frac{q}{2\sqrt{2z_0}})}}{2}$$

$$y_2 = \frac{\sqrt{2z_0} + \sqrt{2z_0 - 4(\frac{p}{2} + z_0 + \frac{q}{2\sqrt{2z_0}})}}{2}$$

$$y_3 = \frac{-\sqrt{2z_0} - \sqrt{2z_0 - 4(\frac{p}{2} + z_0 - \frac{q}{2\sqrt{2z_0}})}}{2}$$

$$y_4 = \frac{-\sqrt{2z_0} + \sqrt{2z_0 - 4(\frac{p}{2} + z_0 - \frac{q}{2\sqrt{2z_0}})}}{2}$$

Of the three roots of the equation, we choose the root equal to $z_3 \approx 3.37672$.

$$y_1 = 0.259$$

$$y_2 = 2.340$$

$$y_3 = -2.340$$

$$y_4 = -0.259$$

$$x_1 = y - \frac{b}{4a} = 2.340 - \frac{-18}{4(9)} = 2.340 + 0.5 = 2.84$$

$$x_2 = y - \frac{b}{4a} = 0.259 - \frac{-18}{4(9)} = 0.259 + 0.5 = 0.759$$

$$x_3 = -2.340 + 0.5 = -1.84$$

$$x_4 = -0.259 + 0.5 = 0.241$$