Answer on Question #46753 – Math – Statistics and Probability

If the moment generating function of X is given by , find c such that

(i) $P[|X| \leq c] = 0.95$
(ii) $P[X \geq c] = 0.025$
(iii) $P[x > c] = 0.5$

Solution:

The moment-generating function is calculated by $M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$. The characteristic function is defined via $\varphi_X(t) = M_{iX}(t) = M_X(it)$. There exists an inversion formula given by

$f_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\varphi_X(t)dt$, where φ_X is integrable characteristic function, f_X is the probability density function.

(i) Using the module definition of absolute value:

$$P[|X| \leq c] = P[-c \leq X \leq c] = P[X \leq c] - P[X \leq -c] = \int_{-c}^{c} f(x)\,dx$$

To get c, we solve this integral and then solve the equation.

$$\int_{-c}^{c} f(x)\,dx = 0.95$$

(ii) $P[X \geq c] = 1 - P[X \leq c] = 1 - \int_{-\infty}^{c} f(x)\,dx$

$$\int_{-\infty}^{c} f(x)\,dx = 1 - 0.025 = 0.975$$

To get c, we solve this integral and then solve the equation.

$$\int_{-\infty}^{c} f(x)\,dx = 0.975$$

(iii) $P[x > c] = \int_{c}^{\infty} f(x)\,dx$

To get c, we solve this integral and then solve the equation.

$$\int_{c}^{\infty} f(x)\,dx = 0.5$$

Answer: (i) $\int_{-c}^{c} f(x)\,dx = 0.95$; (ii) $\int_{-\infty}^{c} f(x)\,dx = 0.975$; (iii) $\int_{c}^{\infty} f(x)\,dx = 0.5$