Answer on Question #46752 – Math – Statistics and Probability

A consumer buys n light bulbs, each of which has a life time that has a mean of 800 hours and a standard deviation of 100 hours. A light bulb is replaced by another as soon as it burns out. Assuming independence of life times, find the smallest value of n so that the succession of light bulbs produces light bulbs for at least 10,000 hours with a probability of 0.9. Do you think it is necessary to know the probability distribution of the light bulbs? Explain.

Solution

Let's use The Central Limit Theorem. Suppose that $X_1, X_2, \dots, X_n$ are independent and all having a distribution $W(\mu; \sigma^2)$. Then if n is sufficiently large:

$$S_n = X_1 + X_2 + \cdots + X_n \approx N(n\mu; n\sigma^2).$$

Now let $X_1, X_2, \dots, X_n$ be the lifetimes of the n bulbs used in succession. Find n such that

$P(X_1 + X_2 + \cdots + X_n > 10000) = 0.9$ and $X_1 + X_2 + \cdots + X_n \mapsto N(800n; 100^2 n)$ hence

$$P(X_1 + X_2 + \cdots + X_n > 10000) = P\left(\frac{X_1 + X_2 + \cdots + X_n - 800n}{100\sqrt{n}} > \frac{100 - 8n}{\sqrt{n}}\right) = P\left(Z > \frac{100 - 8n}{\sqrt{n}}\right)$$
$$= 0.9.$$

From table of standard normal distribution, $\frac{100-8n}{\sqrt{n}} = -1.28$, and $n = 14$ by solving a quadratic equation.

It is not necessary to know the probability distribution of the light bulbs if we can use assumption that n is sufficiently large. Then by The Central Limit Theorem we can work with normal distribution:

$$X_1 + X_2 + \cdots + X_n \mapsto N(800n; 100^2 n).$$