Answer on Question #46697 – Math – Algorithms | Quantitative Methods

Problem.

i) Calculate the third-degree Taylor polynomial about 0 x0 = for 2/1
f (x) = 1(+ x) .

ii) Use the polynomial in part (i) to approximate 1.1 and find a bound for the error involved.

iii) Use the polynomial in part (i) to approximate ∫
+
1.0
0
/1 2
1(x) dx .

Solution:

The question is incorrectly formatted, so we suppose that we have function $f(x)$ and point x_0.
i) The third third-degree Taylor polynomial about x_0 equals

$$P(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3.$$

ii) The approximation of $f(x)$ by the polynomial $P(x)$ in the point a equals

$$P(a) = f(x_0) + \frac{f'(x_0)}{1!}(a - x_0) + \frac{f''(x_0)}{2!}(a - x_0)^2 + \frac{f'''(x_0)}{3!}(a - x_0)^3$$

The error equals $R_3(a) = \frac{f^{(4)}(c)}{4!}(a - x_0)^4$, where c is constant between x_0 and a. Hence to find to find error we need to find maximum and minimum value of the function $\frac{f^{(4)}(c)}{4!}(a - x_0)^4$ (c is variable) on between x_0 and a.

iii) The $\int f(x)dx$ could be approximate with

$$\int_a^b P(x)dx = \int_a^b f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 d(x - x_0)$$

$$= f(x_0)(x - x_0) + \frac{f'(x_0)}{2!}(x - x_0)^2 + \frac{f''(x_0)}{3!}(x - x_0)^3 + \frac{f'''(x_0)}{4!}(x - x_0)^4 \Big|_a^b$$

$$= f(x_0)(b - x_0) + \frac{f'(x_0)}{2!}(b - x_0)^2 + \frac{f''(x_0)}{3!}(b - x_0)^3 + \frac{f'''(x_0)}{4!}(b - x_0)^4$$

$$- \left(f(x_0)(a - x_0) + \frac{f'(x_0)}{2!}(a - x_0)^2 + \frac{f''(x_0)}{3!}(a - x_0)^3 + \frac{f'''(x_0)}{4!}(a - x_0)^4 \right).$$