

Question #46686, Math / Other

Using finite differences, show that the data

x - 3 - 2 -1 0 1 2 3

$f(x)$ 13 7 3 1 1 3 7

represents a second degree polynomial. Obtain this polynomial using interpolation

and find $f(5)$.

Answer.

i	x_i	$f(x_i)$	$D_i^{(1)} = x_i - x_{i+1}$	$D_i^{(2)} = D_i^{(1)} - D_{i+1}^{(1)}$
1	-3	13		
2	-2	7	6	
3	-1	3	4	2
4	0	1	2	2
5	1	1	0	2
6	2	3	-2	2
7	3	7	-4	2

The second differences are constant so the data represents a second degree polynomial.

$$f(x) = ax^2 + bx + c$$

$$f(0) = 1 \rightarrow c = 1;$$

$$f(1) = 1 \rightarrow a + b + 1 = 1 \rightarrow b = -a;$$

$$f(-1) = 3 \rightarrow a - a + 1 = 3 \rightarrow a = 1.$$

Finally: $f(x) = x^2 - x + 1.$

$$f(5.2) = 5.2^2 - 5.2 + 1 = 22.84.$$

Answer provided by <https://www.AssignmentExpert.com>