Answer on Question #46683 – Math – Algorithms | Quantitative Methods

Find all the roots of the equation $x^3 + 6x^2 + 11x + 6 = 0$ by the Graeffe's root squaring method using three squaring.

Solution:

The Graeffe's root squaring method is a direct method to find the roots of any polynomial equation with real coefficients. The basic idea behind this method is to separate the roots of the equations by squaring the roots. This can be done by separating even and odd powers of x in

$$P_n(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1} x + a_n = 0$$

According to our problem we apply this rule for i= 1. We combine on the left and right sides the terms with respect to given rule.

$$x^3 + 11x = -6x^2 - 6$$

$$(x^3 + 11x)^2 = (-6x^2 - 6)^2$$

We obtained the following result.

$$x^6 + 22x^4 + 121x^2 = 36x^4 + 72x^2 + 36$$

Simplify the obtained equation.

$$x^6 + 22x^4 + 121x^2 - 36x^4 - 72x^2 - 36 = x^6 - 14x^4 + 49x^2 - 36$$

Combine terms with same degree.

$$x^6 - 14x^4 + 49x^2 - 36$$

Solve for i =2, the polynomial will be equal.

$$x^3 - 14x^2 + 49x - 36$$

We perform the same operations.

$$x^3 + 49 = 14x^2 + 36$$

$$(x^3 + 49x)^2 = (14x^2 + 36)^2$$

We obtained the following result.

$$x^6 + 98x^4 + 2401x^2 = 196x^4 + 1008x^2 + 1296$$

Combine terms with same degree.

$$x^6 + 98x^4 + 2401x^2 - 196x^4 - 1008x^2 - 1296 = x^6 - 98x^4 + 1393x^2 - 1296$$

Solve for i =3, the polynomial will be equal.

$$x^3 + 1393x = 98x^2 + 1296$$

$$(x^3 + 1393x)^2 = (98x^2 + 1296)^2$$

We obtained the following.

$$x^6 + 2786x^4 + 1\,940\,499x^2 = 9604x^4 + 254016x^2 + 1\,679\,616$$

Combine terms with same degree.

$$x^6 + 2786x^4 + 1\,940\,499x^2 - 9604x^4 - 254016x^2 - 1\,679\,616$$
$$= x^6 - 6818x^4 + 1\,686\,483x^2 - 1\,679\,616$$

Now we can calculate the roots. The roots of polynomial are

$\sqrt{\frac{36}{49}} = 0.85714$ $\sqrt{\frac{49}{14}} = 1.87083,$ $\sqrt{\frac{14}{1}} = 3.74166$

Similarly we find the roots of polynomial when i=2.

$\sqrt[4]{\frac{1296}{1393}} = 0.9821,$ $\sqrt[4]{\frac{1393}{98}} = 1.9417,$ $\sqrt[4]{\frac{98}{1}} = 3.1463$

Finally we calculate the roots of polynomial when i=3.

$\sqrt[8]{\frac{1\,679\,616}{1\,686\,433}} = 0.99949,$ $\sqrt[8]{\frac{1686\,433}{6818}} = 1.99143,$ $\sqrt[8]{\frac{6818}{1}} = 3.01444$