Answer on Question #46682 – Math – Algorithms | Quantitative Methods

The equation 0 2 5

3 2

x + x − = has a positive root in the interval [2,1] . Write a fixed

point iteration method and show that it converges. Starting with initial approximation

5.1 x0 = find the root of the equation. Perform two iterations.

Solution.

$$x^3 + 2x^2 - 5 = 0 \rightarrow x = \sqrt[3]{5 - 2x^2}$$

For the equation $x = g(x)$ **fixed point iteration method is:**

$$x_{n+1} = g(x_n), \;\; n = 0, 1, \dots$$

If $g(x)$ **and** $g'(x)$ **are continuous on an interval J about their root** s **of the equation** $x = g(x)$ **and** $|g'(x)| < 1$ **for all** x **in the interval J then the fixed point iterative process** $x_{n+1} = g(x_n), \;\; n = 0, 1, \dots$ **will converge to the root** $x = s$ **for any initial approximation** x_0 **belongs to the interval J .**

In our case: $g(x) = \sqrt[3]{5 - 2x^2}$**,** $g'(x) = \frac{1}{3}(5 - 2x)^{-\frac{2}{3}}$**,** $g(x)$ **and** $g'(x)$ **are continuous on the interval** $(1, 2)$, $|g'(x)| < \frac{1}{3}$ **on** $(1, 2)$**, thus the iteration method converges.**

$$x_0 = 1.5,$$

$$x_1 = \sqrt[3]{5 - 2 * 1.5^2} = 1.40$$

$$x_2 = \sqrt[3]{5 - 2 * 1.4^2} = 1.45$$

$$x_3 = \sqrt[3]{5 - 2 * 1.45^2} = 1.426 .$$