Answer on Question #46681 – Math – Algebra

Find the number of positive and negative roots of the equation

$$8x^5 + 12x^4 - 10x^3 + 17x^2 - 18x + 5 = 0$$

Solution:

To find possibilities for positive real zeros we count the number of sign changes in the equation for $f(x)$.

This equation has the signs: $+\ +\ -\ +\ -\ +$. It has 2 sign changes, so the equation has 2 or (multiples of 2 less) positive real roots. That is, it has 2 or 0 positive real roots.

To find possibilities for negative real zeros we count the number of sign changes in the equation for $f(-x)$. We obtain this equation by replacing x with $-x$ in the given function.

We can check the number of negative real roots by noting:

$$f(-x) = 8(-x)^5 + 12(-x)^4 - 10(-x)^3 + 17(-x)^2 - 18(-x) + 5$$

We obtained the following result.

$$f(-x) = -8x^5 + 12x^4 + 10x^3 + 17x^2 + 18x + 5$$

This equation has the signs: $-\ +\ +\ +\ +\ +$.

There is only one sign change in this "negative" case, so there is exactly one negative root.

There are 2, or 0 positive real roots, and exactly 1 negative root.

The roots of this equation are: $\frac{-5}{2}, \frac{1}{2}, \frac{1}{2}, -i, i$.