Answer on Question #46672 – Math – Algorithms | Quantitative Methods

Find an interval of unit length which contains the smallest negative root in magnitude of the equation $x^3 + x + 12 = 0$. Taking the midpoint of this interval as initial approximation and using Newton-Raphson method find the root correct to three decimal places.

Solution:

Let $f(x) = x^3 + x + 12 = 0$. Since, the smallest negative real root in magnitude is required; we form a table of values for x < 0.

x	-5	-4	-3	-2	-1
$f(x)$	-118	-56	-18	2	10

Since, f(– 3)f(– 2) < 0, the negative root of smallest magnitude lies in the interval

(– 3, –2). So we put the initial approximation as $x_0 = -2.5$. We have the following.

$$f(x) = x^3 + x + 12 = 0, f'(x) = 3x^2 + 1$$

We use the Newton-Raphson method.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

We have the following results.

$$x_{k+1} = x_k - \frac{x^3{}_k + x_k + 12}{3x^2{}_k + 1} = \frac{2x^3{}_k - 12}{3x^2{}_k + 1}, k = -1, -2 \ldots.$$

We start with $x_0 = -2.5$ and obtained

$$x_1 = \frac{2(-2.5)^3 - 12}{3(-2.5)^2 + 1} = \frac{-43.25}{19.75} = -2.190,$$

$$x_2 = \frac{2(-2.190)^3 - 12}{3(-2.190)^2 + 1} = \frac{-33.007}{15.3883} = -2.145$$

$$x_3 = \frac{2(-2.145)^3 - 12}{3(-2.145)^2 + 1} = \frac{-31.738}{14.803} = -2.144$$

$$x_4 = \frac{2(-2.144)^3 - 12}{3(-2.144)^2 + 1} = \frac{-31.711}{14.790} = -2.144$$

Finally we found that the root correct to three decimal places is x ≈ -2.144.