Problem:

Find the derivative of the function $f(x) = (x^3 + 9x + 4)(10 + \dfrac{1}{x^2})$

Solution

If we have three functions $f(x), g(x), q(x)$ and $f(x) = g(x)q(x)$ then the derivative of the product of functions is calculated by the following formula
$f'(x) = g'(x)q(x) + g(x)q'(x)$.

Use this rule in our problem

Let $g(x) = x^3 + 9x + 4$ and $q(x) = 10 + \dfrac{1}{x^2}$

$$f'(x) = \left((x^3 + 9x + 4)\left(10 + \frac{1}{x^2}\right) \right)' = (x^3 + 9x + 4)'\left(10 + \frac{1}{x^2}\right) + (x^3 + 9x + 4)\left(10 + \frac{1}{x^2}\right)' =$$

$$= (3x^2 + 9)\left(10 + \frac{1}{x^2}\right) + (x^3 + 9x + 4)\left(-2\frac{1}{x^3}\right) = 30x^2 + 90 + 3 + \frac{9}{x^2} - 2 - \frac{18}{x^2} - \frac{8}{x^3} =$$

$$= 30x^2 + 91 - \frac{9}{x^2} - \frac{8}{x^3}$$

Answer

$$f'(x) = 30x^2 + 91 - \frac{9}{x^2} - \frac{8}{x^3}$$