Answer on Question #46562 – Math – Calculus

b) The Fibonacci sequence is defined as follows:

$a_{n+2} = a_n + a_{n+1} \quad for \quad n \geq 1, \; where \quad a_1 = a_2 = 1.$

i. List the first eight terms of the sequence.

ii. Find $Lim(a_n + 1/a_n)$ assuming that it exists.

Solution:

I

First eight terms of the Fibonacci sequence:

$$1, 1, 2, 3, 5, 8, 13, 21$$

II

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

As previously stated, the Fibonacci Sequence is defined as:

$$a_{n+1} = a_n + a_{n-1}$$

Therefore, the ratio of consecutive terms of this sequence is defined as:

$$\frac{a_{n+1}}{a_n} = \frac{a_n + a_{n-1}}{a_n}$$

Well, $\frac{a_n + a_{n-1}}{a_n} = \frac{a_n}{a_n} + \frac{a_{n-1}}{a_n} = 1 + \frac{a_{n-1}}{a_n}$

L,The limiting value of the the ratio of consecutive terms of the Fibonacci Sequence, must be equal to the defined value of the ratio. In other words,

$$L = \frac{a_{n+1}}{a_n}$$

Plugging this back into our equation for the ratio of the consecutive terms, we get $\frac{a_{n+1}}{a_n} = L = 1 + \frac{1}{L}$. Because all terms a_n are positive, the limit L is also positive.

Using the quadratic equation $L^2 - L - 1 = 0$ to solve for L, we find that

$$L = \frac{1+\sqrt{5}}{2} \approx 1.62$$

Answer:

I) First eight terms of the Fibonacci sequence:

$$1, 1, 2, 3, 5, 8, 13, 21$$

II) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1+\sqrt{5}}{2} \approx 1.62.$