Question:

A ball is dropped from a height of 10m. Each time the ball strikes the ground it bounces vertically to a height that is 3/4 of the preceding height. Find the total distance the ball will travel if it is allowed to bounce indefinitely.

Solution:

When the ball hits the ground the first time it has travelled D_1= 10m.

Between the first and second bounces the ball will travel

$$D_2 = \left(\frac{3}{4}\right)(10) + \left(\frac{3}{4}\right)(10)$$

Between the second and third bounces the ball will travel

$$D_3 = \left(\frac{3}{4}\right)^2 (20)$$

and so on, so between the $(n-1)^{th}$ and n^{th} bounces the ball will travel

$$D_n = \left(\frac{3}{4}\right)^{n-1} (20)$$

Summing these distances (after D_1) we have

$$\text{distance} = 10 + \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (20)$$

Now this is almost the geometric series, we know

$$\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n (20) = \frac{20}{1 - 3/4} = 80$$

and we can see that

$$\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n (20) = 20 + \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (20) = 80$$

Therefore the sum we want is 60. Hence

$$\text{distance} = 10 + \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (20) = 10 + 60 = 70 \text{ m}$$

Answer: 70 m.