Answer on Question #46501 – Math - Statistics and Probability

Problem.

A population of size 500 is divided into 4 strata. The following table gives the (5) data on size and standard deviation of each stratum.

	STRATA			
	I	II	III	IV
Size, N_i	100	150	150	100
S.D., σ_i	5	8	7	10

A stratified random sample of size 100 is to be drawn from the population.
Determine the size of samples from each of these strata for:
i) proportional allocation
ii) Neyman's optimal allocation

Solution:

(i) Strata sample sizes are determined by the following equation for proportional allocation:

$$n_i = \frac{N_i}{N} \cdot n$$

where n_i is the sample size for stratum i, N_i is the population size for stratum i ($N_1 = 100$, $N_2 = 150, N_3 = 150, N_4 = 100$), $N = 500$ is total population size, and $n = 100$ is total sample size.
Hence for proportional allocation and random sample of size 100 we will obtain the following table:

	STRATA			
	I	II	III	IV
Size, n_i	20	30	30	20

(ii) Strata sample sizes are determined by the following equation for Neyman's optimal allocation:

$$n_i = \frac{N_i \sigma_i}{\sum_i N_i \sigma_i} \cdot n$$

where n_i is the sample size for stratum i, N_i is the population size for stratum i ($N_1 = 100$, $N_2 = 150, N_3 = 150, N_4 = 100$), σ_i is the standard deviation of stratum i ($\sigma_1 = 5, \sigma_2 = 8$, $\sigma_3 = 7, \sigma_4 = 10$) and $n = 100$ is total sample size.
Hence for Neyman's optimal allocation and random sample of size 100 we will obtain the following table:

	STRATA			
	I	II	III	IV
Size, n_i	$13\frac{1}{3} \approx 13$	32	28	$26\frac{2}{3} \approx 27$