Answer on Question #46412 – Math – Trigonometry
Problem

1. Prove $(1-\cos A)(1-\sec A) = \tan A \sin A$

2. Prove $\sec A(1-\sin A)(\sec A + \tan A) = 1$

Solution

1. $(1-\cos A)(1-\sec A) = \tan A \sin A$

   You know that

   - $\sec A = \dfrac{1}{\cos A}$

   - $\sin^2 A + \cos^2 A = 1$

   - $\tan A = \dfrac{\sin A}{\cos A}$

   Rewrite the left-hand side

$$(1-\cos A)(1-\sec A) = (1-\cos A)\left(1 - \frac{1}{\cos A}\right) = (1-\cos A)\frac{(\cos A - 1)}{\cos A}$$

Rewrite the right-hand side

$$\tan A \sin A = \frac{\sin A}{\cos A}\sin A = \frac{\sin^2 A}{\cos A} = \frac{1-\cos^2 A}{\cos A} = \frac{(1-\cos A)(1+\cos A)}{\cos A}$$ So we can see

that

$$(1-\cos A)\frac{(\cos A - 1)}{\cos A} \neq \frac{(1-\cos A)(1+\cos A)}{\cos A}$$

If you change the sign "-" on the "+" sign in the second bracket then you will get
the correct statement $(1-\cos A)(1+\sec A) = \tan A \sin A$.

2. $\sec A(1-\sin A)(\sec A + \tan A) = 1$

   Rewrite the left-hand side

$$\sec A(1-\sin A)(\sec A + \tan A) = \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) =$$

$$= \frac{1}{\cos A}(1-\sin A)\left(\frac{1+\sin A}{\cos A}\right) = \frac{1}{\cos^2 A}(1-\sin^2 A) = \frac{\cos^2 A}{\cos^2 A} = 1$$