

Answer on Question #46340 – Math – Vector Calculus

Problem.

For the vectors $\vec{a} = xyz\vec{i} - 2xz^2\vec{j} + xz\vec{k}$ and $\vec{b} = 2z\vec{i} + y\vec{j} - x\vec{k}$

Find $\frac{\partial^2}{\partial x \partial y} (\vec{a} \times \vec{b})$ at $(2, 0, -1)$

Remark: I suppose that the statement is incorrectly formatted. The correct statement is:

“For the vectors $\vec{a} = xyz\vec{i} - 2xz^2\vec{j} + xz\vec{k}$ and $\vec{b} = 2z\vec{i} + y\vec{j} - x\vec{k}$

Find $\frac{\partial^2}{\partial x \partial y} (\vec{a} \times \vec{b})$ at $(2, 0, -1)$ ”

Solution:

$$\vec{a} \times \vec{b} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ xyz & -2xz^2 & xz \\ 2z & y & -x \end{bmatrix} = (2x^2z^2 - xyz)\vec{i} + (x^2yz + 2z^2x)\vec{j} + (y^2xz + 4z^3x)\vec{k}$$

$$\frac{\partial^2}{\partial x \partial y} (\vec{a} \times \vec{b}) = -z\vec{i} + 2xz\vec{j} + 2yz\vec{k}$$

For $(x, y, z) = (2, 0, -1)$ we have $\frac{\partial^2}{\partial x \partial y} (\vec{a} \times \vec{b})(2, 0, -1) = \vec{i} - 4\vec{j} + 0\vec{k} = (1, -4, 0)$.

Answer: $\vec{i} - 4\vec{j} + 0\vec{k} = (1, -4, 0)$.