Answer on Question #46307 – Math - Other

A flower vase, in the form of a hexagonal prism, is to be filled with 512 cubic inches of water. Find the height of the water if the wet portion of the flower vase and its volume are numerically equal.

Solution:

A hexagonal prism is a prism composed of two hexagonal bases and six rectangular sides. The regular right hexagonal prism of edge length L has volume equal:

$$V = \frac{3}{2}\sqrt{3}L^2 h$$

In our task we need to find the height of the wet portion of the flower vase, we can suppose that this mean he base and the inner sides of the prism.

We put the sides of the hexagon be length L, and its height be h. The area of regular hexagon is equal:

$$\text{Area} = \frac{3\sqrt{3}}{2}L^2$$

Also we can note the perimeter around the prism, which is equal to 6L. Thus we can construct a formula for determination the area of the wet portion of the flower vase.

$$\text{Area}_{\text{wet portion}} = 6Lh + \frac{3\sqrt{3}}{2}L^2$$

According to the condition of the task we know that area of the wet portion of the flower vase and its volume are numerically equal. In this case we can write the equity.

$$\frac{3}{2}\sqrt{3}L^2 h = 6Lh + \frac{3\sqrt{3}}{2}L^2$$

Simplify obtained equation by dividing both sides of the equation by $\frac{3}{2}\sqrt{3}L$.

$$Lh = 6h \cdot \frac{2}{3\sqrt{3}} + L$$

Simplify the equation.

$$Lh = \frac{4h}{\sqrt{3}} + L$$

From the found equation we need to determine the value of L. We subtract all terms from the left side of the equation.

$$Lh - \frac{4h}{\sqrt{3}} - L = 0$$

L we take out the parenthesis.

$$\mathrm{L}(\mathrm{h}-1)-\frac{4\mathrm{h}}{\sqrt{3}}=0$$

Then we add $\frac{4\mathrm{h}}{\sqrt{3}}$ to both sides of the equation.

$$\mathrm{L}(\mathrm{h}-1)=\frac{4\mathrm{h}}{\sqrt{3}}$$

Now we divide both sides of the equation by $(\mathrm{h}-1)$. We obtained the following result.

$$\mathrm{L}=\frac{4\mathrm{h}}{\sqrt{3}}\cdot\frac{1}{(\mathrm{h}-1)}=\frac{4\mathrm{h}}{\sqrt{3}(\mathrm{h}-1)}$$

As we know from the given condition that hexagonal prism, is to be filled with 512 cubic inches of water, this mean that we can substitute the value of volume into the formula to find the value of the height h of water in the vase.

$$\frac{3}{2}\sqrt{3}\mathrm{L}^2\mathrm{h}=512$$

Also we have found the equation for L, so we can substitute into the formula noted above.

$$\frac{3}{2}\sqrt{3}\mathrm{h}\,\left(\frac{4\mathrm{h}}{\sqrt{3}(\mathrm{h}-1)}\right)^2=512$$

Firstly we simplify the expression in the parenthesis.

$$\frac{3}{2}\sqrt{3}\mathrm{h}\,\left(\frac{16\mathrm{h}^2}{3(\mathrm{h}^2-2\mathrm{h}+1)}\right)=512$$

$$\frac{\sqrt{3}\mathrm{h}}{1}\cdot\left(\frac{8\mathrm{h}^2}{(\mathrm{h}^2-2\mathrm{h}+1)}\right)=512$$

$$\frac{8\sqrt{3}\mathrm{h}^3}{(\mathrm{h}^2-2\mathrm{h}+1)}=512$$

Now we multiply both sides of the equation by the denominator $(\mathrm{h}^2-2\mathrm{h}+1)$.

$$8\sqrt{3}\mathrm{h}^3=512(\mathrm{h}^2-2\mathrm{h}+1)$$

Simplify by opening parenthesis in right side of the equation.

$$8\sqrt{3}\mathrm{h}^3=512\mathrm{h}^2-1024\mathrm{h}+512$$

$$8\sqrt{3}\mathrm{h}^3-512\mathrm{h}^2+1024\mathrm{h}-512=0$$

Now we can divide all terms by 8.

$$\sqrt{3}h^3 - 64h^2 + 128h - 64 = 0$$

After mathematical operations we find the following real roots:

$$h_1 \approx 0.867 \text{ inches}$$

$$h_2 \approx 1.222 \text{ inches}$$

$$h_3 \approx 34.861 \text{ inches}$$

If we consider the first root $h_1 \approx 0.867$ into the formula

$$L = \frac{4h}{\sqrt{3}(0.867 - 1)} = \frac{4 \cdot 0.867}{\sqrt{3}(-0.139)}$$

So, we can conclude that value of $h_1 \approx 0.867$ inches is not convenient for us because it gives a negative length value.

Then we check another root $h_2 \approx 1.222$ inches, substitute into the formula for length.

$$L = \frac{4h}{\sqrt{3}(1.222 - 1)} = \frac{4 \cdot 1.222}{\sqrt{3}(0.222)} \approx 12.712 \text{ inches}$$

The value we obtained cannot be the dimension of our vases. We reject this value.

Finally we consider the last root, which is equal to $h_3 \approx 34.861$ inches. Substitute into the formula for determination the length.

$$L = \frac{4 \cdot 34.861}{\sqrt{3}(34.861 - 1)} = \frac{139.444}{58.6489724} \approx 2.378 \text{ inches}$$

Now we can calculate the area of the wet portion of the flower vase to check the found values.

$$\text{Area}_{\text{wet portion}} = 6 \cdot (2.378) \cdot (34.861) + \frac{3\sqrt{3}}{2}(2.378)^2$$

Simplify the expression.

$$\text{Area}_{\text{wet portion}} = 497.397 + 14.691 \approx 512.09 \text{ inches}^2$$

Answer: The height of the water is equal to 34.861 inches.