Answer on Question #46303 – Math - Trigonometry

If $\sin(A) = \frac{3}{5}$, and $\sin(B) = \frac{5}{13}$, where A and B are acute angles. Find $\cos(A)\cos(B) + \sin(A)\sin(B)$

Solution

Use the basic relationship between the sine and the cosine (the Pythagorean Identity):

$$sin^2x + cos^2x = 1$$

To calculate $cos(A)$

$$sin^2(A) + cos^2(A) = 1$$

$$cos^2(A) = 1 - sin^2(A)$$

$$\cos(A) = \pm\sqrt{1 - sin^2(A)}$$

Here we have only "plus" because the A angle is acute angle.

$$\cos(A) = \sqrt{1 - sin^2(A)}$$

Substitute $\sin(A) = \frac{3}{5}$ and simplify

$$cosA = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

The same for $cos(B)$

$$sin^2B + cos^2B = 1$$

$$cos^2B = 1 - sin^2B$$

$$cosB = \pm\sqrt{1 - sin^2B}$$

Take only $\cos(B) = \sqrt{1 - sin^2B}$ because the B angle is acute angle.

Substitute $\sin(B) = \frac{5}{13}$ and simplify

$$\cos(B) = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{169 - 25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

So

$$\cos(A)\cos(B) + \sin(A)\sin(B) = \frac{4}{5} * \frac{12}{13} + \frac{3}{5} * \frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$

Answer: $\frac{63}{65}$