Question:

find the exact value of $\cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right)$. For full credit, explain your reasoning.

Solution:

We know that

$$\sin^2\alpha + \cos^2\alpha = 1.$$

Therefore,

$$\cos\alpha = \sqrt{1 - \sin^2\alpha}\,.$$

But in our case the angle α is

$$\alpha = \sin^{-1}\left(\frac{1}{4}\right).$$

Using the identity that

$$\sin(\sin^{-1}(x)) = x.$$

We get

$$\cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right) = \sqrt{1 - \sin^2\left(\sin^{-1}\left(\frac{1}{4}\right)\right)} = \sqrt{1 - \frac{1}{4^2}} = \frac{\sqrt{15}}{4}.$$

Answer:

$$\cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right) = \frac{\sqrt{15}}{4}.$$