Answer on Question #46276 – Math – Other

A container trough is in the form of a triangular prism whose opposite sides are equilateral triangles with side 2 feet and is 4 feet apart.

a. Find the height of the triangle.

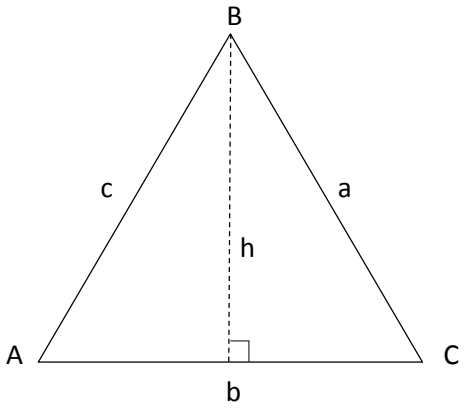
b. Find the volume of the trough.

c. If 2 cubic feet of water is placed inside the trough, what is the level of the water?

d. Find the area of the wet portion.

Solution:

Start to consider our problem with the definition. We have a prism with the cross section of a triangle. We know that the opposite sides are equilateral triangles. An equilateral triangle is a special case of a triangle where all 3 sides have equal length and all 3 angles are equal to 60 degrees. The altitude shown h is hb or, the altitude of b. For equilateral triangles h = ha = hb = hc.



Based on the above information, we can note the following: if we know the length of a side = 2 feet, we can calculate the height h of the triangle. The altitude h of an equilateral triangle is calculated by

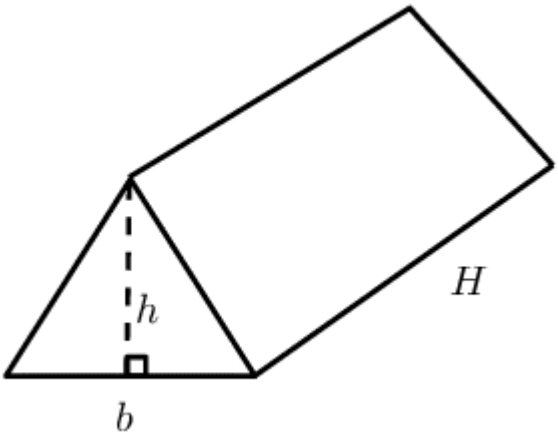
$$\mathrm{h} = \mathrm{a} \cdot \sin 60° = \frac{1}{2}\sqrt{3} \cdot 2 = \sqrt{3} = 1.73205 \text{ feet}$$

Then we can find the area of a triangle. The area will be equal to

$$\text{Area} = \frac{1}{2} \cdot \mathrm{a} \cdot \mathrm{h} = \frac{1}{2} \cdot 2 \cdot \sqrt{3} = \sqrt{3} = 1.73205 \text{ feet}^2$$

In order to find the volume of the trough, we need to find the volume of a triangular prism, which is calculated by the formula.

$$\text{Volume} = \text{Area} \cdot \text{Length}$$



We have length of prism, which is equal to 4 feet. So we can find the volume of the given figure. Substitute the values into the formula.

$$\text{Volume } = \sqrt{3} \cdot 4 = 4\sqrt{3} = 6.928\ feet^3$$

Then our task is to find the level of the water if 2 cubic feet of water is placed inside the trough.

First we define the area that will occupy the predetermined water level, as we know in the form of a cut across our gutters is an equilateral triangle. So we can write the following.

$$\frac{2\ \text{feet}^3}{4\ \text{feet}} = \frac{1}{2}\text{feet}^2$$

We have found the area of required equilateral triangle. Now we can calculate the value of the level that we need to find out. Substitute into the formula of the area, where x is the side length, so the area is

$$\text{Area} = \frac{1}{4}\sqrt{3}x^2$$

Now we substitute the value of area of required equilateral triangle.

$$\frac{1}{4}\sqrt{3}\text{x}^2 = \frac{1}{2}$$

Simplify our equation to find the value of x.

$$\frac{\sqrt{3}\text{x}^2}{4} = \frac{1}{2}$$

$$2\sqrt{3}\text{x}^2 = 4$$

Divide both sides of the equation by $2\sqrt{3}$, we obtained the following.

$$\text{x}^2 = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Finally the value of x will be equal.

$$x = \sqrt{\frac{2}{\sqrt{3}}}$$

Now we substitute the find value into the formula to find the height of water.

$$\mathrm{h} = \mathrm{a} \cdot \sin 60° = \frac{1}{2} \cdot \sqrt{3} \cdot \mathrm{x}$$

$$\mathrm{h} = \frac{1}{2} \cdot \sqrt{\frac{2}{\sqrt{3}}} \cdot \sqrt{3} = 0.93057 \text{ feet}$$

Then we can calculate the Area of wet portion.

$$\text{Area of wet portion} = (2 \cdot \frac{1}{2}) + (2 \cdot \sqrt{\frac{2}{\sqrt{3}}} \cdot 4)$$

Simplify the expression in the parenthesis.

$$\text{Area of wet portion} = 1 + (8.59656) = 9.59656 \text{ feet}^2.$$