Answer on Question #46256 – Math – Statistics and Probability

Question.

i) Suppose that the life of a certain type of electron tube has an exponential distribution with a mean life of 500 hours. Find the probability that it will last for another 600 hours if the tube has been in operation for 300 hours.

ii) For a random variable $X, E(X) = 10$, and $Var(X) = 25$ find the positive values of a and b such that $Y = aX - b$ has expectation zero and variance 1.

Solution.

i) The density of exponential distribution has the next form: $f_X(x) = \begin{cases} 0, & x \le 0 \\ \lambda e^{-\lambda x}, & x > 0 \end{cases}, E(X) = \frac{1}{\lambda}$.

In our case $\frac{1}{\lambda} = 500 \Rightarrow \lambda = \frac{1}{500} \Rightarrow f_X(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{500} e^{-\frac{x}{500}}, & x > 0 \end{cases}$. We shall find the required probability by the next formula: $P = \frac{P(X \ge 900)}{P(X \ge 300)} = \frac{\frac{1}{500}\int_{900}^{+\infty} e^{-\frac{xdx}{500}}}{\frac{1}{500}\int_{300}^{+\infty} e^{-\frac{xdx}{500}}} = \frac{\int_{900}^{+\infty} e^{-\frac{xdx}{500}}}{\int_{300}^{+\infty} e^{-\frac{xdx}{500}}}$.

$$\int_{900}^{+\infty} e^{-\frac{x}{500}} dx = -500 e^{-\frac{x}{500}} \Big|_{900}^{+\infty} = 0 + 500 e^{-\frac{9}{5}} = 500 e^{-\frac{9}{5}}.$$

$$\int_{300}^{+\infty} e^{-\frac{x}{500}} dx = -500 e^{-\frac{x}{500}} \Big|_{300}^{+\infty} = 0 + 500 e^{-\frac{3}{5}} = 500 e^{-\frac{3}{5}}.$$

$$P = \frac{500 e^{-\frac{9}{5}}}{500 e^{-\frac{3}{5}}} = e^{-\frac{6}{5}}.$$

ii) $E(Y) = E(aX - b) = E(aX) - E(b) = aE(X) - b = 10a - b = 0 \Rightarrow b = 10a.$

$$Var(Y) = Var(aX - b) = Var(aX) = a^2 Var(X) = 25a^2 = 1 \Rightarrow a = \frac{1}{5} \Rightarrow b = \frac{10}{5} = 2.$$

Answer.

- **i)** $e^{-\frac{6}{5}}$
- **ii)** $a = \frac{1}{5}, b = 2.$