

Answer on Question #46251 – Math – Statistics and Probability

Question.

In a test on electric bulbs, it was found that the life time of a particular brand was normally distributed with an average life of 2000 hours and S.D. of 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for **(i)** more than 2100 hours, **(ii)** less than 1950 hours and **(iii)** between 1900 and 2100 hours.

Solution.

Let ξ be the life time of a particular brand. Then $\eta = \frac{\xi - 2000}{60}$ has a standard normal distribution i. e. $P(\eta < x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$. $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$ is a tabulated function of Laplace.

$$\text{(i)} \quad P(\xi > 2100) = 1 - P(\xi < 2100) = 1 - P\left(\frac{\xi - 2000}{60} < 1.67\right) = 1 - (0.5 + \Phi(1.67)) =$$

$= 1 - (0.5 + 0.45254) = 0.04746$. The corresponding number of bulbs is $2500 \cdot 0.04746 \approx 119$ bulbs.

$$\text{(ii)} \quad P(\xi < 1950) = P\left(\frac{\xi - 2000}{60} < -0.83\right) = 0.5 - \Phi(0.83) = 0.5 - 0.29673 = 0.20327. \quad \text{The corresponding number of bulbs is } 2500 \cdot 0.20327 \approx 508 \text{ bulbs.}$$

$$\text{(iii)} \quad P(1900 < \xi < 2100) = P\left(-1.67 < \frac{\xi - 2000}{60} < 1.67\right) = 2 \cdot \Phi(1.67) = 0.90508. \quad \text{The corresponding number of bulbs is } 2500 \cdot 0.90508 \approx 2263 \text{ bulbs.}$$

Answer.

(i) 119 bulbs

(ii) 508 bulbs

(iii) 2263 bulbs