Answer on Question #46250 – Math – Statistics and Probability

Question. The joint probability function of two discrete variables X and Y is given by

$f(x,y) = c(2x+y)$, where x and y can assume all integral values such that $0 \le x \le 2$, $0 \le y \le 3$, and $f(x,y) = 0$ otherwise. Find

(i) The value of the constant c

(ii) $P(X = 2, Y = 1)$

(iii) $P(X \ge 1, Y \le 2)$

(iv) Marginal probability functions of X and Y. Check whether X and Y are independent.

Solution.

(i) Use the next property of $f(x,y)$: $\sum_{i=0}^{2}\sum_{j=0}^{3} f(i,j) = 1$. We have:

$$\sum_{i=0}^{2}\big(f(i,0)+f(i,1)+f(i,2)+f(i,3)\big) = f(0,0)+f(0,1)+f(0,2)+f(0,3)+$$

$$+f(1,0)+f(1,1)+f(1,2)+f(1,3)+f(2,0)+f(2,1)+f(2,2)+f(2,3) =$$

$$= 0\cdot c + c + 2c + 3c + 2c + 3c + 4c + 5c + 4c + 5c + 6c + 7c = 42c = 1 \Rightarrow c = \frac{1}{42}.$$

(ii) $f(x,y) = \frac{1}{42}(2x+y)$. $P(X=2, Y=1) = f(2,1) = \frac{1}{42}\cdot 5 = \frac{5}{42}$.

(iii) $P(X \ge 1, Y \le 2) = \sum_{i=1}^{2}\sum_{j=0}^{2} f(i,j) = \sum_{i=1}^{2}\big(f(i,0)+f(i,1)+f(i,2)\big) = f(1,0)+$

$$+f(1,1)+f(1,2)+f(2,0)+f(2,1)+f(2,2) = \frac{1}{42}(2+3+4+4+5+6) = \frac{4}{7}.$$

(iv) $f_X(x) = \sum_{j=0}^{3} f(x,j) = f(x,0)+f(x,1)+f(x,2)+f(x,3) =$

$$= \frac{1}{42}(2x+2x+1+2x+2+2x+3) = \frac{1}{42}(8x+6) = \frac{1}{21}(4x+3), 0 \le x \le 2.$$

$$f_Y(y) = \sum_{i=0}^{2} f(i,y) = f(0,y)+f(1,y)+f(2,y) = \frac{1}{42}(y+2+y+4+y) =$$

$$= \frac{1}{42}(3y+6) = \frac{1}{14}(y+2), 0 \le y \le 3.$$

Since $f_X(x)f_Y(y) = \left(\frac{4}{21}x+\frac{1}{7}\right)\left(\frac{1}{14}y+\frac{1}{7}\right) = \frac{2}{147}xy+\frac{4}{147}x+\frac{1}{98}y+\frac{1}{49} \ne \frac{1}{21}x+\frac{1}{42}y = f(x,y)$, X and Y are not independent.

Answer.

(i) $c = \frac{1}{42}$

(ii) $P(X=2, Y=1) = \frac{5}{42}$

(iii) $P(X \geq 1, Y \leq 2) = \frac{4}{7}$

(iv) $f_X(x) = \frac{1}{21}(4x + 3), 0 \leq x \leq 2$

$f_Y(y) = \frac{1}{14}(y + 2), 0 \leq y \leq 3.$

X and Y are not independent.