Answer on Question #46203 – Math - Differential Calculus | Equations

Question:

The surface of a ball of radius A is kept at a temperature zero. If the initial temperature in the ball is f (r), write down the boundary conditions and show that the temperature in the ball at time t, u (r, t), is the solution to the equation:

$$c^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}\right)=\frac{\partial u}{\partial t}$$

Solution:

Boundary conditions:

1) The surface of a ball of radius A is kept at a temperature zero:
$$T|_{r=A}=0$$
2) The initial temperature in the ball is f (r):
$$T|_{t=0}=f(r)$$

The heat equation describes the distribution of heat (or variation in temperature) in a given region over time:

$$\Delta u=\frac{1}{c^2}\frac{\partial u}{\partial t}$$

For spherical coordinates:

$$\Delta f=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$

If $\frac{\partial u}{\partial\theta}=0$ and $\frac{\partial u}{\partial\phi}=0$:

$$\Delta u=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)=\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}$$

Therefore:

$$\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}=\frac{1}{c^2}\frac{\partial u}{\partial t}$$

$$c^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}\right)=\frac{\partial u}{\partial t}$$