Answer on Question #46201 – Math - Calculus

Question:

Compute the Jacobian matrices using the chain rule for $z = u^2 + v^2$

where $u = 2x + 7,\ v = 3x + y + 7.$

Solution.

We see that z depends on u and v, but u and v, in turn, depend on x and y. It means that z depends on x and y too.

Hence we have Jacobian matrix $J = \frac{\partial(z)}{\partial(x,y)} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)$.

Chain rule for the computing of this Jacobian matrix is the product of two matrices

$$J = \frac{\partial(z)}{\partial(x,y)} = \frac{\partial(z)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)}$$

where Jacobian matrix

$$\frac{\partial(u.v)}{\partial(x,y)} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$$

For given two functions
$u(x,y) = 2x + 7$
$v(x,y) = 3x + y + 7$
we obtain the Jacobian matrix

$$\frac{\partial(u.v)}{\partial(x,y)} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

For the function $z = u^2 + v^2$

we have $\frac{\partial(z)}{\partial(u,v)} = \left(\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}\right) = (2u, 2v)$

The product of two matrices is

$$J = \frac{\partial(z)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = (2u, 2v) \cdot \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} = (4u + 6v, 2v) =$$
$$= \big(4(2x+7) + 6(3x+y+7), 2(3x+y+7)\big) =$$
$$= (26x + 6y + 70, 6x + 2y + 14)$$

Answer: Jacobian matrices

$$J = \frac{\partial(z)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = (26x + 6y + 70, 6x + 2y + 14)$$