

Answer on Question #46194 – Mathematics – Calculus

Question:

Find the domain of the following functions $f(x,y)$ and evaluate $f(3,2)$.

1) $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$;

2) $f(x,y) = x \ln(y^2-x)$.

Solution:

Definition. A function of two variables is a function whose domain is a subset of the plane R^2 and whose range is a subset of R . If we denote the domain set by D , then a function f is a rule that assigns to every point $(x,y) \in D$ a real number $(x,y) \in R$.

1) $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$.

To define the assumed domain D of this rational function, we need to find all points for which the function is valid. On the one hand, the radicand cannot be negative inside the square root, so we require that $x + y + 1 \geq 0$. On the other hand, we cannot divide by zero, so for the denominator of the function we require that $x - 1 \neq 0$. Therefore the assumed domain specified by the system:

$$\begin{cases} x + y + 1 \geq 0 \\ x - 1 \neq 0. \end{cases} \quad (1)$$

Let's rewrite the system in the obvious form:

$$\begin{cases} y \geq -(x + 1) \\ x \neq 1. \end{cases} \quad (2)$$

Now, let's graph our solution

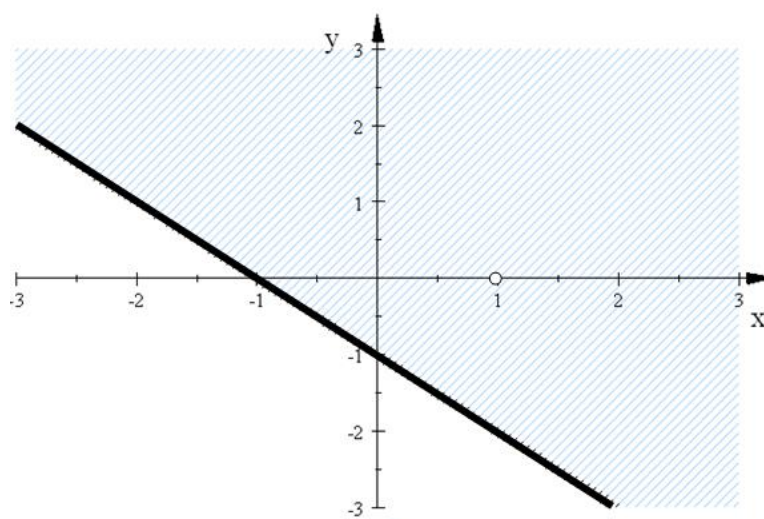


Fig.1

As we see in fig.1, the domain is the half plane $y \geq -(x + 1)$ with the exception of point $x=1$.

Let's evaluate $f(3,2)$:

$$f(3,2) = f(x,y)|_{(3,2)} = \left(\frac{\sqrt{x+y+1}}{x-1} \right) |_{(3,2)} = \frac{\sqrt{3+2+1}}{3-1} = \frac{\sqrt{6}}{2}.$$

2) $f(x,y) = x \ln(y^2-x)$.

The argument of the logarithmic function is strictly positive, so the assumed domain requires that

$y^2 - x > 0$. Let's rewrite this inequality in the following form: $x < y^2$. It is easy to see, that the domain D is the outer part of parabola $x = y^2$, which is symmetric with respect to the x -axis (fig.2). Note, that the curve $x = y^2$ does not belong to the domain.

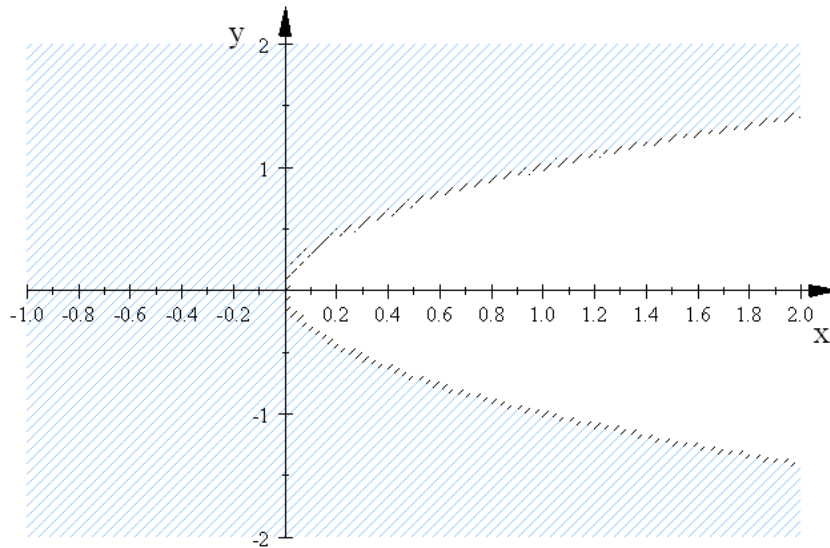


Fig.2

Let's evaluate $f(3,2)$:

$$f(3,2) = f(x,y)|_{(3,2)} = x \ln(y^2 - x) = 3 \ln(2^2 - 3) = 3 \ln 1 = 0.$$

Answer:

1) The domain D is defined by the system $\begin{cases} y \geq -(x+1) \\ x \neq 1. \end{cases}$, $f(3,2) = \frac{\sqrt{6}}{2}$.

2) The domain D is defined by the inequality $x < y^2$, $f(3,2) = 0$.